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THE UNIVERSITY OF ALBERTA  
INVESTIGATION INTO THE PROPERTIES OF DIELECTRIC  
DISKS COATED WITH NONMAGNETIC METAL FOR  
MICROWAVE TUBE APPLICATIONS

BY



S.N. HANNA

A THESIS  
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THE UNIVERSITY OF ALBERTA  
FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "Investigation into the Properties of Dielectric Disks Coated with Nonmagnetic Metal for Microwave Tube Applications" submitted by Saad N. Hanna in partial fulfilment of the requirements for the degree of Master of Science.





## ABSTRACT

Dielectric disks can be used to load a waveguide or a resonant cavity. In this work the properties of dielectric disks coated on both sides with a thin layer of nonmagnetic metal has been investigated. A cylindrical guide periodically loaded with these disks has been analysed by two methods. In the first method a periodic structure consisting of up to four regions has been considered. By allotting appropriate values of permittivity, the coated disk can be considered as three regions of this structure. In the second method, it was assumed that the metallic layer is so thin that it does not affect the basic field pattern. The losses arising from the surface current in this metallic layer have been calculated.

One section of the loaded waveguide, if shorted on both sides, can be used as a resonant cavity. For this case the effect of coating on the Q factor has been found.

Measurement of the Q factor provides an accurate method of determining the effective resistance of the coating. It has been also used to check the theoretical assumptions.



## ACKNOWLEDGEMENTS

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## INTRODUCTION

In a travelling wave tube <sup>(1)</sup>, an axial electron beam with a velocity small compared with the velocity of light, interacts with an electromagnetic wave. A slow wave structure capable of propagating the electromagnetic wave over the desired frequency band is used to reduce the phase velocity of the wave to a value approximately equal to the velocity of the electron beam. Although the slow wave structure commonly used is the helix, other structures as dielectric loaded waveguide or corrugated guide can also be used.

The input and output couplers cannot be matched over the whole frequency band so that at the output terminal the reflected wave provides a mechanism for feed back and possible oscillation. A travelling wave tube has a high gain so that very little mismatch is sufficient to produce oscillation. Therefore some sort of isolation between the input and the output of the slow wave structure has to be provided. It is a usual practice to introduce an attenuator near the middle of the travelling wave tube, so that the loss in the backward direction is approximately equal to the gain in the forward direction. This attenuator affects the forward wave, but at this point appreciable modulation of the beam has taken place which allows a build up of the electromagnetic wave in the second half of the structure, which is amplified and appears at the output. In certain types of structures a lossy wire is placed in the





middle to obtain the required attenuation.

In this work, the properties of a dielectric disk coated on both sides with a thin layer of metal has been investigated to determine the possibility of using such disks to obtain the required attenuation in a travelling wave tube. This method is applicable when the propagating structure is a waveguide periodically loaded with dielectric disks.



## CHAPTER I

### WAVE PROPAGATION IN CIRCULAR GUIDE PERIODICALLY LOADED WITH DIELECTRIC DISKS

In this chapter a periodic structure consisting of a waveguide loaded with dielectric disks is analysed. The analysis has then been extended to the cases where the loading element is a disk consisting of up to three layers of different dielectric materials.

The relation between the phase shift per section and the structure dimensions has been found in each case. This has been done by writing the solution of Maxwell's equations for each region separately and then matching the field components at the various boundaries. This technique becomes extremely cumbersome when the disk consists of more than three layers. In addition the entire process must be repeated if the number of layers is changed. The wave matrices method may be used to deal with stratified dielectric loaded structures<sup>(2,3)</sup>, but this approach has not been considered here.

#### 1.1 Periodic Structure with Two Regions

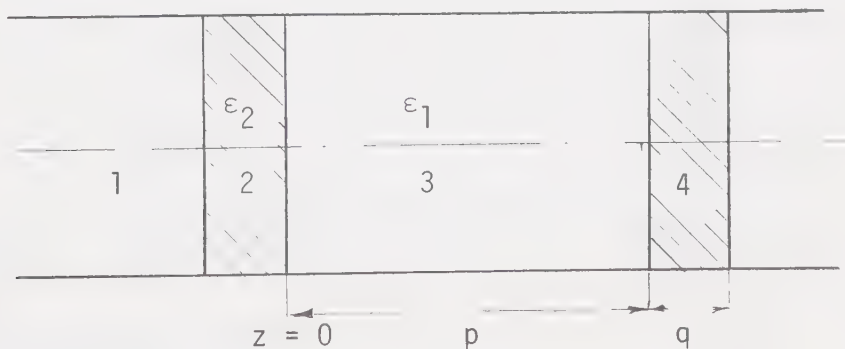


Fig. 1.1 Periodic Structure with Two Regions.



a)  $E_{n,1}$  modes

A circular guide, loaded with dielectric disks is shown in figure 1.1, where the dielectric disks are denoted by the shaded areas.

The solution of Maxwell's equations for an  $E_{n,1}$  mode in cylindrical waveguide may be written, in terms of the forward waves (A and C) and the backward waves (B and D), for the dielectric and air regions. These equations are:

$$\begin{aligned}
 E_z &= (A e^{-j\beta_2 z} + B e^{+j\beta_2 z}) J_n(\chi r) \cos n\phi \\
 E_r &= \frac{j\beta_2}{\chi} (A e^{-j\beta_2 z} - B e^{+j\beta_2 z}) J'_n(\chi r) \sin n\phi \\
 E_\phi &= \frac{-j\beta_2 n}{\chi^2 r} (A e^{-j\beta_2 z} - B e^{+j\beta_2 z}) J_n(\chi r) \sin n\phi \quad \dots 1.1 \\
 H_r &= \frac{j\omega\epsilon_2 n}{\chi^2 r} (A e^{-j\beta_2 z} + B e^{+j\beta_2 z}) J_n(\chi r) \sin n\phi \\
 H_\phi &= \frac{j\omega\epsilon_2}{\chi} (A e^{-j\beta_2 z} + B e^{+j\beta_2 z}) J'_n(\chi r) \cos n\phi
 \end{aligned}$$

for the dielectric region, and

$$\begin{aligned}
 E_z &= (C e^{-j\beta_1 z} + D e^{+j\beta_1 z}) J_n(\chi r) \cos n\phi \\
 E_r &= \frac{j\beta_1}{\chi} (C e^{-j\beta_1 z} - D e^{+j\beta_1 z}) J'_n(\chi r) \cos n\phi \\
 E_\phi &= \frac{-j\beta_1 n}{\chi^2 r} (C e^{-j\beta_1 z} - D e^{+j\beta_1 z}) J_n(\chi r) \sin n\phi \quad \dots 1.2 \\
 H_r &= \frac{j\omega\epsilon_1 n}{\chi^2 r} (C e^{-j\beta_1 z} + D e^{+j\beta_1 z}) J_n(\chi r) \sin n\phi \\
 H_\phi &= \frac{j\omega\epsilon_1}{\chi} (C e^{-j\beta_1 z} + D e^{+j\beta_1 z}) J'_n(\chi r) \cos n\phi
 \end{aligned}$$

for the air region, where

$$\beta^2 = \omega^2 \mu \epsilon - \chi^2$$

$$\chi = \frac{P_{n,1}}{a}, \text{ where } P_{n,1} \text{ is the } 1^{\text{th}} \text{ root of } J_n(x_a) = 0$$





According to Floquet's theorem, the field components in region (4) of figure 1.1 is given by equation 1.1 multiplied by  $e^{-j\psi}$  where  $\psi$  is the phase change per period. Matching the transverse field components at the boundaries  $z = 0$  and  $z = p$  we get:

$$\beta_2 A - \beta_2 B - \beta_1 C + \beta_1 D = 0 \quad \dots (a)$$

$$\epsilon_2 A + \epsilon_2 B - \epsilon_1 C - \epsilon_1 D = 0$$

$$\beta_2 A e^{+j(2\theta_2 - \psi)} - \beta_2 B e^{-j(2\theta_2 + \psi)} - \beta_1 C e^{-2j\theta_1} + \beta_1 D e^{2j\theta_1} = 0 \quad \dots (c)$$

$$\epsilon_2 A e^{+j(2\theta_2 - \psi)} + \epsilon_2 B e^{-j(2\theta_2 + \psi)} - \epsilon_1 C e^{-2j\theta_1} - \epsilon_1 D e^{2j\theta_1} = 0 \quad \dots (d)$$

..... 1.3

where  $2\theta_1$  is the phase change in the air region,  $\beta_1 p$ , and  $2\theta_2$  is the phase change in the dielectric region,  $\beta_2 q$ .

Multiplying (b) and (c) in equation 1.3 by  $\omega$  and using the expression for the wave impedance of the  $E_{n,1}$  mode,  $Z = \frac{\beta}{\omega\epsilon}$ , we get:

$$\frac{\beta_2 A}{Z_2} + \frac{\beta_2 B}{Z_2} - \frac{\beta_1 C}{Z_1} - \frac{\beta_1 D}{Z_1} = 0 \quad \dots (e)$$

$$\frac{\beta_2}{Z_2} A e^{+j(2\theta_2 - \psi)} + \frac{\beta_2}{Z_2} B e^{-j(2\theta_2 + \psi)} - \frac{\beta_1}{Z_1} C e^{-2j\theta_1} - \frac{\beta_1}{Z_1} D e^{2j\theta_1} = 0 \quad (f)$$

For equations (a), (c), (e) and (f) to have a unique solution,

$$\begin{vmatrix} \beta_2 & -\beta_2 & -\beta_1 & -\beta_1 \\ \frac{\beta_2}{Z_2} & \frac{\beta_2}{Z_2} & \frac{-\beta_1}{Z_1} & \frac{-\beta_1}{Z_1} \\ \beta_2 e^{j(2\theta_2 - \psi)} & -\beta_2 e^{-j(2\theta_2 + \psi)} & -\beta_1 e^{-2j\theta_1} & -\beta_1 e^{2j\theta_1} \\ \frac{\beta_2}{Z_2} e^{j(2\theta_2 - \psi)} & -\frac{\beta_2}{Z_2} e^{-j(2\theta_2 + \psi)} & -\frac{\beta_1}{Z_1} e^{-2j\theta_1} & -\frac{\beta_1}{Z_1} e^{2j\theta_1} \end{vmatrix} = 0$$



Dividing columns (1) and (2) by  $\beta_2$  and  $-\beta_2$  and columns (3) and (4) by  $\beta_1$  and  $-\beta_1$  respectively we get:

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ \frac{1}{Z_2} & \frac{-1}{Z_2} & \frac{-1}{Z_1} & \frac{-1}{Z_1} \\ e^{j(2\theta_2-\psi)} & e^{-j(2\theta_2+\psi)} & -e^{-2j\theta_1} & -e^{2j\theta_1} \\ \frac{e^{j(2\theta_2-\psi)}}{Z_2} & \frac{-e^{-j(2\theta_2+\psi)}}{Z_2} & \frac{-e^{-2j\theta_1}}{Z_1} & \frac{e^{2j\theta_1}}{Z_1} \end{vmatrix} = 0 \quad \dots\dots\dots 1.4$$

Expanding and solving for  $\psi$  gives

$$\cos \psi = \cos 2\theta_1 \cos 2\theta_2 - \frac{1}{2} \left( \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} \right) \sin 2\theta_1 \sin 2\theta_2 \quad \dots\dots\dots 1.5$$

b)  $H_{n,1}$  modes

The field components that may exist in the dielectric and air regions for this mode are:

$$\begin{aligned} H_z &= (A e^{-j\beta_2 z} + B e^{j\beta_2 z}) J_n(\chi r) \cos n\phi \\ H_r &= \frac{j\beta_2}{\chi} (A e^{-j\beta_2 z} - B e^{j\beta_2 z}) J'_n(\chi r) \cos n\phi \\ H_\phi &= \frac{-j\beta_2 n}{\chi^2 r} (A e^{-j\beta_2 z} - B e^{j\beta_2 z}) J_n(\chi r) \sin n\phi \quad \dots\dots\dots 1.6 \\ E_r &= \frac{-j\omega\mu n}{\chi^2 r} (A e^{-j\beta_2 z} + B e^{j\beta_2 z}) J_n(\chi r) \sin n\phi \\ E_\phi &= \frac{-j\omega\mu}{\chi} (A e^{-j\beta_2 z} + B e^{j\beta_2 z}) J'_n(\chi r) \cos n\phi \end{aligned}$$

for the dielectric region, and





$$\begin{aligned}
H_z &= (C e^{-j\beta_1 z} + D e^{j\beta_1 z}) J_n(\chi r) \cos n\phi \\
H_r &= \frac{j\beta_1}{\chi} (C e^{-j\beta_1 z} - D e^{j\beta_1 z}) J'_n(\chi r) \cos n\phi \\
H_\phi &= \frac{-j\beta_1 n}{\chi^2 r} (C e^{-j\beta_1 z} - D e^{j\beta_1 z}) J_n(\chi r) \sin n\phi \quad \dots\dots 1.7 \\
E_r &= \frac{-j\omega\mu n}{\chi^2 r} (C e^{-j\beta_1 z} + D e^{j\beta_1 z}) J_n(\chi r) \sin n\phi \\
E_\phi &= \frac{-j\omega\mu}{\chi} (C e^{-j\beta_1 z} + D e^{j\beta_1 z}) J'_n(\chi r) \cos n\phi
\end{aligned}$$

for the air region.

Matching the transverse field components at  $z = 0$  and at  $z = p$  and using Floquet's theorem for the field in region (4) we get:

$$\begin{aligned}
A + B - C - D &= 0 \quad \dots\dots(a) \\
-\beta_2 A + \beta_2 B + \beta_1 C - \beta_1 D &= 0 \quad \dots\dots(b) \\
Ae^{j(2\theta_2 - \psi)} + Be^{-j(2\theta_2 + \psi)} - Ce^{-2j\theta_1} - De^{2j\theta_1} &= 0 \quad \dots\dots(c) \\
-\beta_2 Ae^{j(2\theta_2 - \psi)} + \beta_2 Be^{-j(2\theta_2 + \psi)} + \beta_1 Ce^{-2j\theta_1} - \beta_1 De^{2j\theta_1} &= 0 \quad \dots\dots(d) \\
&\dots\dots 1.8
\end{aligned}$$

Dividing (b) and (d) in 1.10 by  $\omega\mu$  and using the expression for the wave impedance of the  $H_{n,1}$  mode,  $Z = \frac{\omega\mu}{\beta}$ , we get:

$$\begin{aligned}
\frac{A}{Z_2} + \frac{B}{Z_2} - \frac{C}{Z_1} - \frac{D}{Z_1} &= 0 \quad \dots\dots(e) \\
\frac{Ae^{j(2\theta_2 - \psi)}}{Z_2} + \frac{Be^{-j(2\theta_2 + \psi)}}{Z_2} - \frac{Ce^{-2j\theta_1}}{Z_1} - \frac{De^{2j\theta_1}}{Z_1} &= 0 \quad \dots\dots(f)
\end{aligned}$$

For equations (a), (c), (e) and (f) to have a unique solution,



$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ \frac{1}{Z_2} & \frac{-1}{Z_2} & \frac{-1}{Z_1} & \frac{1}{Z_1} \\ e^{j(2\theta_2-\psi)} & e^{-j(2\theta_2+\psi)} & -e^{-2j\theta_1} & -e^{2j\theta_1} \\ \frac{e^{j(2\theta_2-\psi)}}{Z_2} & \frac{-e^{-j(2\theta_2+\psi)}}{Z_2} & \frac{-e^{-2j\theta_1}}{Z_1} & \frac{e^{2j\theta_1}}{Z_1} \end{vmatrix} = 0$$

.... 1.9

The determinant 1.9 is similar to 1.4, therefore the solution is given by equation 1.5.

Below the cut-off frequency of the air region and above the cut-off frequency of the dielectric, the structure may still propagate. In this case  $\beta_1$  becomes imaginary and may be replaced by <sup>(4)</sup>.

$$-j\gamma = \sqrt{\chi^2 - \omega^2 \mu \epsilon_1}$$

Equation 1.5 then becomes

$$\cos \psi = \cosh \gamma p \cos 2\theta_2 + \frac{1}{2} \left( \frac{\gamma \epsilon_r}{\beta_2} - \frac{\beta_2}{\gamma \epsilon_r} \right) \sinh \gamma p \sin 2\theta_2$$

.... 1.10

for the  $E_{n,1}$  mode, where  $\epsilon_r = \frac{\epsilon_2}{\epsilon_1}$ , and

$$\cos \psi = \cosh \gamma p \cos 2\theta_2 - \frac{1}{2} \left( \frac{\beta_2}{\gamma} - \frac{\gamma}{\beta_2} \right) \sinh \gamma p \sin 2\theta_2$$

.... 1.11

for the  $H_{n,1}$  mode.



## 1.2 Periodic Structure with Three Regions.

A periodic structure with three regions is as shown in figure 1.2. The same technique used in section 1.1 can be applied for this case. The solution of Maxwell's equations, in terms of

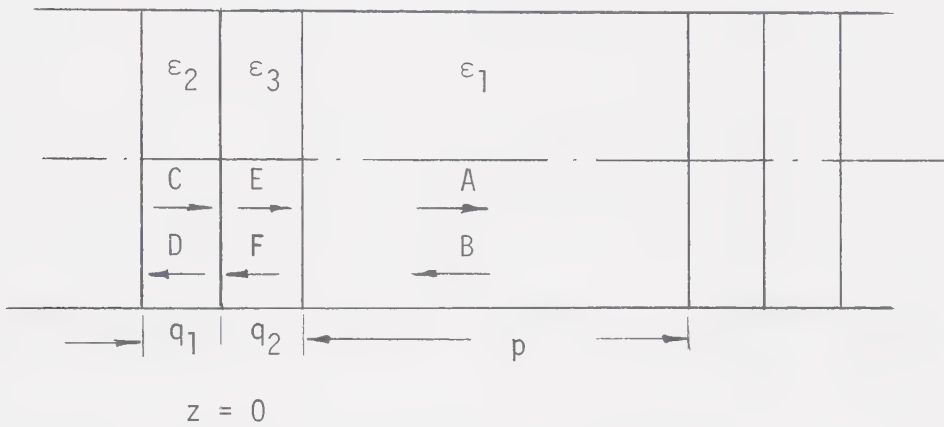


Fig. 1.2 Periodic Structure with Three Regions.

forward waves (A, C and E) and backward waves (B, D and F), may be written for each region. Matching the transverse field components at the boundaries we get:

$$\beta_2 C - \beta_2 D - \beta_3 E + \beta_3 F = 0 \quad \dots (a)$$

$$\epsilon_2 C + \epsilon_2 D - \epsilon_3 E - \epsilon_3 F = 0 \quad \dots (b)$$

$$\beta_3 E e^{-j\beta_3 q_2} - \beta_3 F e^{j\beta_3 q_2} - \beta_1 A e^{-j\beta_1 q_2} + \beta_1 B e^{j\beta_1 q_2} = 0 \quad \dots (c)$$

$$\epsilon_3 E e^{-j\beta_3 q_2} + \epsilon_3 F e^{j\beta_3 q_2} - \epsilon_1 A e^{-j\beta_1 q_2} - \epsilon_1 B e^{j\beta_1 q_2} = 0 \quad \dots (d)$$

$$\beta_1 A e^{-j\beta_1 (q_2 + p)} - \beta_1 B e^{j\beta_1 (q_2 + p)} - \beta_2 C e^{j(\beta_2 q_1 - \psi)} + \beta_2 D e^{-j(\beta_2 q_1 + \psi)} = 0 \quad \dots (e)$$





$$\epsilon_1 A e^{-j\beta_1(q_2+p)} + \epsilon_1 B e^{j\beta_1(q_2+p)} - \epsilon_2 C e^{j(\beta_2 q_1 - \psi)} - \epsilon_2 D e^{-j(\beta_2 q_1 + \psi)} = 0$$

...(f)

....1.12

Multiplying (b), (d) and (f) by  $\omega$  and substituting  $Z = \frac{\beta}{\omega \epsilon}$ ,

the following determinantal equation can be obtained:

$$\begin{vmatrix} 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & \frac{1}{Z_2} & \frac{1}{Z_2} & \frac{-1}{Z_3} & \frac{-1}{Z_3} \\ -e^{-j\beta_1 q_2} & e^{j\beta_1 q_2} & 0 & 0 & e^{-j\beta_3 q_2} & e^{j\beta_3 q_2} \\ \frac{-e^{-j\beta_1 q_2}}{Z_1} & \frac{-e^{j\beta_1 q_2}}{Z_1} & 0 & 0 & \frac{e^{-j\beta_3 q_2}}{Z_3} & \frac{e^{j\beta_3 q_2}}{Z_3} \\ e^{-j\beta_1(q_2+p)} & -e^{j\beta_1(q_2+p)} & -e^{j(\beta_2 q_1 - \psi)} & e^{-j(\beta_2 q_1 + \psi)} & 0 & 0 \\ \frac{e^{-j\beta_1(q_2+p)}}{Z_1} & \frac{e^{j\beta_1(q_2+p)}}{Z_1} & \frac{-e^{j(\beta_2 q_1 - \psi)}}{Z_2} & \frac{-e^{-j(\beta_2 q_1 + \psi)}}{Z_2} & 0 & 0 \end{vmatrix} = 0$$

Expanding and solving for  $\psi$  we get:

$$\begin{aligned} \cos \psi = & \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \\ & - \frac{1}{2} \left( \frac{Z_1}{Z_3} + \frac{Z_3}{Z_1} \right) \sin 2\theta_1 \cos 2\theta_2 \sin 2\theta_3 \\ & - \frac{1}{2} \left( \frac{Z_2}{Z_3} + \frac{Z_3}{Z_2} \right) \cos 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \\ & - \frac{1}{2} \left( \frac{Z_2}{Z_1} + \frac{Z_1}{Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \cos 2\theta_3 \end{aligned}$$

.... 1.13

where  $2\theta_1$ ,  $2\theta_2$  and  $2\theta_3$  are the phase changes in each region.



The same result may be obtained by matching the transverse field components for the  $H_{n,1}$  mode.

If  $2\theta_1$  or  $2\theta_3$  equal zero in equation 1.13 this case yields a structure with two regions and equation 1.5 is obtained.

### 1.3 Periodic Structure with Four Regions.

The same technique of solving Maxwell's equations in each region and matching the field components at the boundaries was carried out and the following result was obtained:

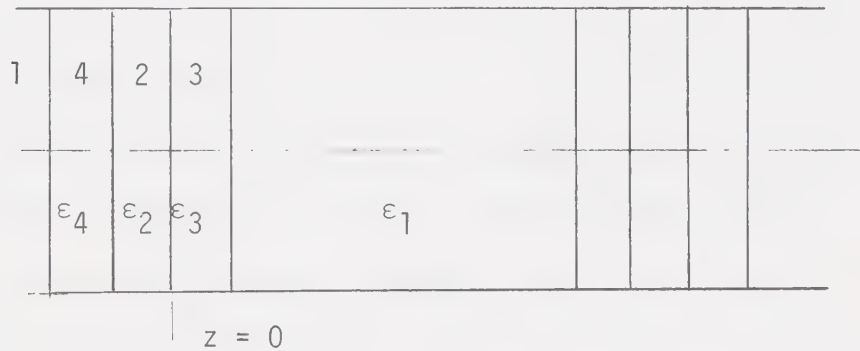


Fig. 1.3 Periodic Structure with Four Regions.

$$\begin{aligned}
 \cos \psi = & \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \\
 & - \frac{1}{2} \left( \frac{Z_2}{Z_1} + \frac{Z_1}{Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \\
 & - \frac{1}{2} \left( \frac{Z_1}{Z_4} + \frac{Z_4}{Z_1} \right) \sin 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \sin 2\theta_4 \\
 & - \frac{1}{2} \left( \frac{Z_3}{Z_1} + \frac{Z_1}{Z_3} \right) \sin 2\theta_1 \cos 2\theta_2 \sin 2\theta_3 \cos 2\theta_4
 \end{aligned}$$



$$\begin{aligned}
& - \frac{1}{2} \left( \frac{Z_3}{Z_4} + \frac{Z_4}{Z_3} \right) \cos 2\theta_1 \cos 2\theta_2 \sin 2\theta_3 \sin 2\theta_4 \\
& - \frac{1}{2} \left( \frac{Z_2}{Z_3} + \frac{Z_3}{Z_2} \right) \cos 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \cos 2\theta_4 \\
& - \frac{1}{2} \left( \frac{Z_2}{Z_4} + \frac{Z_4}{Z_2} \right) \cos 2\theta_1 \sin 2\theta_2 \cos 2\theta_3 \sin 2\theta_4 \\
& + \frac{1}{2} \left( \frac{Z_1 Z_2}{Z_3 Z_4} + \frac{Z_3 Z_4}{Z_1 Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \sin 2\theta_4
\end{aligned}$$

.... 1.14

#### 1.4 Case of Disks Similarly Coated on Both Sides.

This section deals with dielectric disks coated on both sides with another dielectric material. The coatings are of equal thickness.

This case is a special case of the four regions periodic structure discussed in the previous section. Equation 1.14 can be applied with  $2\theta_3 = 2\theta_4$  and  $Z_3 = Z_4$ . Substituting, we get:

$$\begin{aligned}
\cos \psi &= \cos 2\theta_1 \cos 2\theta_2 \cos 4\theta_3 \\
& - \frac{1}{2} \left( \frac{Z_2}{Z_1} + \frac{Z_1}{Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \cos^2 2\theta_3 \\
& + \frac{1}{2} \left( \frac{Z_1 Z_2}{Z_3^2} + \frac{Z_3^2}{Z_1 Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \sin^2 2\theta_3 \\
& - \left( \frac{Z_2}{Z_3} + \frac{Z_3}{Z_2} \right) \cos 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \cos 2\theta_3 \\
& - \left( \frac{Z_1}{Z_3} + \frac{Z_3}{Z_1} \right) \sin 2\theta_1 \cos 2\theta_2 \sin 2\theta_3 \cos 2\theta_3
\end{aligned}$$

.... 1.15





Equation 1.15 may be applied to the case where a disk is coated to improve its surface properties, i.e. prevention of electric breakdown or chemical decomposition under adverse conditions.



## CHAPTER II

### WAVE PROPAGATION IN A PERIODIC STRUCTURE LOADED WITH DIELECTRIC DISKS COATED WITH NONMAGNETIC METAL

In this chapter, wave propagation in a cylindrical waveguide, periodically loaded with different type of disks has been investigated. The dielectric disks considered have a very thin surface coating of nonmagnetic metal. To prevent excessive attenuation and thus permit appreciable propagation, the thickness of the coating should only be a small fraction of the skin depth.

Two different approaches have been used. The first one consists in appropriately modifying the result obtained in section 1.4. In the second approach, the metallic film is considered to be very thin so that attenuation and phase change through this layer can be neglected. However the surface current generated in the coating, by the transverse electric field, causes a discontinuity in the H field. Matching the field components at the boundaries gives us a solution which is found to be in good agreement with the first one.

#### 2.1 Solution Derived From Section 1.4.

When the dielectric disks are coated with metal, the solution can be obtained by replacing the values of  $\beta$  and  $\epsilon$  in regions 3 and 4 of figure 1.6 by the corresponding values for the metal.



These values are<sup>(5)</sup>:

$$\epsilon = \frac{-j\sigma}{\omega} \quad \dots 2.1$$

$$\beta = (1-j)\sqrt{\frac{\omega\mu\sigma}{2}} \quad \dots 2.2$$

$$\frac{\beta}{\epsilon} = (1+j)\omega\sqrt{\frac{\mu}{2\sigma}} \quad \dots 2.3$$

Where  $\sigma$  is the bulk conductivity of the metal.

In equation 1.15,  $2\theta_3 = \beta_3 q_3$  where  $q_3$  is the thickness of the metal which is considered to be a fraction of a skin depth, therefore:

$$\cos 2\theta_3 \approx 1$$

$$\sin 2\theta_3 \approx 2\theta_3 \quad \dots 2.4$$

Substituting 2.1, 2.3 and 2.4 in equation 1.15 we get:

$$\begin{aligned} \cos\psi = & \cos 2\theta_1 \cos 2\theta_2 \\ & - \frac{1}{2} \left( \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + \sigma_1^2 Z_1 Z_2 - \frac{q_3^2 \mu^2 \omega^2}{Z_1 Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \\ & + \left( j \sigma_1 Z_2 - \frac{q_3 \mu \omega}{Z_2} \right) \cos 2\theta_1 \sin 2\theta_2 \\ & + \left( j \sigma_1 Z_1 - \frac{q_3 \mu \omega}{Z_1} \right) \sin 2\theta_1 \cos 2\theta_2 \quad \dots 2.5 \end{aligned}$$

where  $\sigma_1 = \sigma q_3$ .



In this case  $\psi$  is a complex constant,

$$\psi = (\phi - j\alpha)$$

where  $\alpha$  is the attenuation per section and  $\phi$  is the phase shift per section.

Therefore,  $\cos\psi = \cos(\phi - j\alpha)$

$$= \cos\phi \cosh\alpha + j \sin\phi \sinh\alpha \quad \dots\dots 2.6$$

Equating the real and imaginary parts of equations 2.5 and 2.6., we get:

$$\sin\phi \sinh\alpha = \sigma_1 Z_2 \cos 2\theta_1 \sin 2\theta_2 + \sigma_1 Z_1 \sin 2\theta_1 \cos 2\theta_2 \quad \dots\dots 2.7$$

and,

$$\begin{aligned} \cos\phi \cosh\alpha &= \cos 2\theta_1 \cos 2\theta_2 \\ &- \frac{1}{2} \left( \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + \sigma_1^2 Z_1 Z_2 - \frac{q_3^2 \mu^2 \omega^2}{Z_1 Z_2} \right) \sin 2\theta_1 \sin 2\theta_2 \\ &- \omega \mu q_3 \left( \frac{\cos 2\theta_1 \sin 2\theta_2}{Z_2} + \frac{\sin 2\theta_1 \cos 2\theta_2}{Z_1} \right) \end{aligned} \quad \dots\dots 2.8$$

Equating the left hand side of equations 2.7 and 2.8 to T and S respectively we get:

$$\sin\phi \sinh\alpha = T \quad \dots\dots 2.7a$$

$$\cos\phi \cosh\alpha = S \quad \dots\dots 2.8a$$

Solving for  $\phi$ , which must be a real constant, we get:

$$\begin{aligned} \sin\phi &= \frac{\pm \left( \sqrt{\frac{(S^2 + T^2 - 1)^2}{4} + T^2} - \frac{(S^2 + T^2 - 1)}{2} \right)^{\frac{1}{2}}}{\pm V} \quad \dots\dots 2.9 \end{aligned}$$





The attenuation per section  $\alpha$  is a positive number and so  $\cosh \alpha$  and  $\sinh \alpha$  are positive. The sign of T and S in equations 2.7a and 2.8a defines the value of  $\phi$  as shown in table 2.1. The attenuation  $\alpha$  can be found by substituting the value of  $\phi$  in equation 2.7a and is given by:

$$\alpha = \sinh^{-1} \frac{T}{V} \quad \dots 2.10$$

## 2.2 Approximate Solution.

It was mentioned earlier that the thickness of the metallic coating should be a small fraction of the skin depth. If the coating is very thin the only effect of this layer is to give rise to a surface current density (J) in phase with the transverse electric field  $E_r$ .

If  $\sigma_1$  is the conductivity of the metallic layer per unit area of the coating, then

$$\bar{J} = \sigma_1 \bar{E}_r$$

The effect of this current is to cause a discontinuity in the transverse component of the magnetic field. Using equations 1.1 and 1.2 and matching the transverse field components at  $z = 0$  in fig. 1.1:

$$E_{r2} = E_{r3}$$



$$\therefore \beta_2 A - \beta_2 B - \beta_1 C + \beta_1 D = 0$$

and

$$H_{\phi 2} - \sigma_1 E_{r2} = H_{\phi 3}$$

$$\therefore (\omega \epsilon_2 - \sigma_1 \beta_2) A + (\omega \epsilon_2 + \sigma_1 \beta_2) B - \omega \epsilon_1 C - \omega \epsilon_1 D = 0$$

Matching the transverse field components at  $z = p$  and using Floquet's theorem we get:

$$E_{r3} = E_{r4}$$

$$\therefore \beta_2 A e^{j(2\theta_2 - \psi)} - \beta_2 B e^{-j(2\theta_2 + \psi)} - \beta_1 C e^{-2j\theta_1} + \beta_1 D e^{2j\theta_1} = 0$$

and

$$H_{\phi 3} - \sigma_1 E_{r3} = H_{\phi 4}$$

$$\begin{aligned} \therefore (\omega \epsilon_1 - \sigma_1 \beta_1) C e^{-2j\theta_1} + (\omega \epsilon_1 + \sigma_1 \beta_1) D e^{2j\theta_1} - \omega \epsilon_2 A e^{j(2\theta_2 - \psi)} \\ - \omega \epsilon_2 B e^{-j(2\theta_2 + \psi)} = 0 \end{aligned}$$

where  $2\theta_1$ ,  $2\theta_2$  and  $\psi$  have the same meaning as before.

For these four equations to have a unique solution:

$$\begin{vmatrix} \beta_2 & -\beta_2 & -\beta_1 & \beta_1 \\ \beta_2 \left( \frac{1}{Z_2} - \sigma_1 \right) & \beta_2 \left( \frac{1}{Z_2} + \sigma_1 \right) & \frac{-\beta_1}{Z_1} & \frac{-\beta_1}{Z_1} \\ \beta_2 e^{j(2\theta_2 - \psi)} & -\beta_2 e^{-j(2\theta_2 + \psi)} & -\beta_1 e^{-2j\theta_1} & \beta_1 e^{2j\theta_1} \\ \frac{\beta_2}{Z_2} e^{j(2\theta_2 - \psi)} & \frac{\beta_2}{Z_2} e^{-j(2\theta_2 + \psi)} & -\beta_1 \left( \frac{1}{Z_1} - \sigma_1 \right) e^{-2j\theta_1} & -\beta_1 \left( \frac{1}{Z_1} + \sigma_1 \right) e^{2j\theta_1} \end{vmatrix} = 0$$



Expanding and solving for  $\psi$  we get:

$$\begin{aligned}\cos \psi &= \cos 2\theta_1 \cos 2\theta_2 \\ &- \frac{1}{2} \left( \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + \sigma_1^2 Z_1 Z_2 \right) \sin 2\theta_1 \sin 2\theta_2 \\ &+ j\sigma_1 (Z_2 \cos 2\theta_1 \sin 2\theta_2 + Z_1 \sin 2\theta_1 \cos 2\theta_2) \\\dots 2.11\end{aligned}$$

In this case

$$\sin \phi \sinh \alpha = \sigma_1 (Z_2 \cos 2\theta_1 \sin 2\theta_2 + Z_1 \sin 2\theta_1 \cos 2\theta_2) \\\dots 2.12$$

$$\begin{aligned}\cos \phi \cosh \alpha &= \cos 2\theta_1 \cos 2\theta_2 \\ &- \frac{1}{2} \left( \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + \sigma_1^2 Z_1 Z_2 \right) \sin 2\theta_1 \sin 2\theta_2 \\\dots 2.13\end{aligned}$$

The solution for  $\alpha$  and  $\phi$  can be obtained from equation 2.9, 2.10 and table 2.1, by substituting the appropriate values for T and S given by equations 2.12 and 2.13.

To compare equations 2.5 and 2.11, a structure which will be discussed in chapter 5 is considered. This structure has the dimensions:

$$a = .02 \text{ meter}$$

$$q = .0024 \text{ meter}$$

$$p = .14 \text{ meter}$$

$$\epsilon_r = 93.5$$





The frequency at the  $\Pi$  mode was found to be 5.75 G Hz

$$\therefore \beta_1 = 5.55 \qquad \beta_2 = 1158$$

If we consider the coating to be an Aluminum layer of thickness  $200^\circ \text{ \AA}$ , then

$$\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} = 2.679$$

$$\frac{q_3^2 \mu^2 \omega^2}{Z_1 Z_2} = 3.02 \times 10^{-8}$$

$$\omega \mu q_3 = 7.21 \times 10^{-4}$$

These figures show that terms multiplied by  $q_3$  and  $q_3^2$  in equation 2.5 are much smaller than the other terms. Therefore when the metallic layer thickness, is of the order of a few hundreds Angstroms, equations 2.12 and 2.13 hold with reasonable accuracy.

Below the cut-off frequency of the air region, and for the  $E_{m,n}$  mode, equations 2.12 and 2.13 become:

$$\sin \phi \sinh \alpha = \sigma_1 (Z_2 \cosh \gamma p \sin 2\theta_2 - \frac{\omega \gamma}{\epsilon_1} \sinh \gamma p \cos 2\theta_2)$$

$$\begin{aligned} \cos \phi \cosh \alpha &= \cosh \gamma p \cos 2\theta_2 \\ &+ \frac{1}{Z} \left( \frac{\gamma \epsilon_r}{\beta_2} - \frac{\beta_2}{\gamma \epsilon_r} + \sigma_1^2 \frac{\omega \gamma Z_2}{\epsilon_1} \right) \sinh \gamma p \sin 2\theta_2 \end{aligned}$$

.... 2.14



where  $-j\gamma = \sqrt{\chi^2 - \omega^2 \mu \epsilon_1}$

Calculation of  $\alpha$  and  $\phi$  for the previously mentioned structure shows that the shape of the dispersion curve is affected by the losses.

Shamaly <sup>(7)</sup> has shown that where losses are present in a structure the group velocity in the stop bands does not equal zero.

Curves of  $\phi$  and  $\alpha$  vs  $\frac{a}{\lambda}$  for two different structures and for different values of layer conductivity  $\sigma_1$ , are shown in figures 2.1 to 2.6.

| S   | T   | $\phi$             | $\phi$              | V   |
|-----|-----|--------------------|---------------------|-----|
| +ve | +ve | $0 < \phi < 90$    | $\sin^{-1} V$       | +ve |
| -ve | +ve | $90 < \phi < 180$  | $180 - \sin^{-1} V$ | +ve |
| -ve | -ve | $180 < \phi < 270$ | $180 + \sin^{-1} V$ | -ve |
| +ve | -ve | $270 < \phi < 360$ | $360 - \sin^{-1} V$ | -ve |

Table 2.1



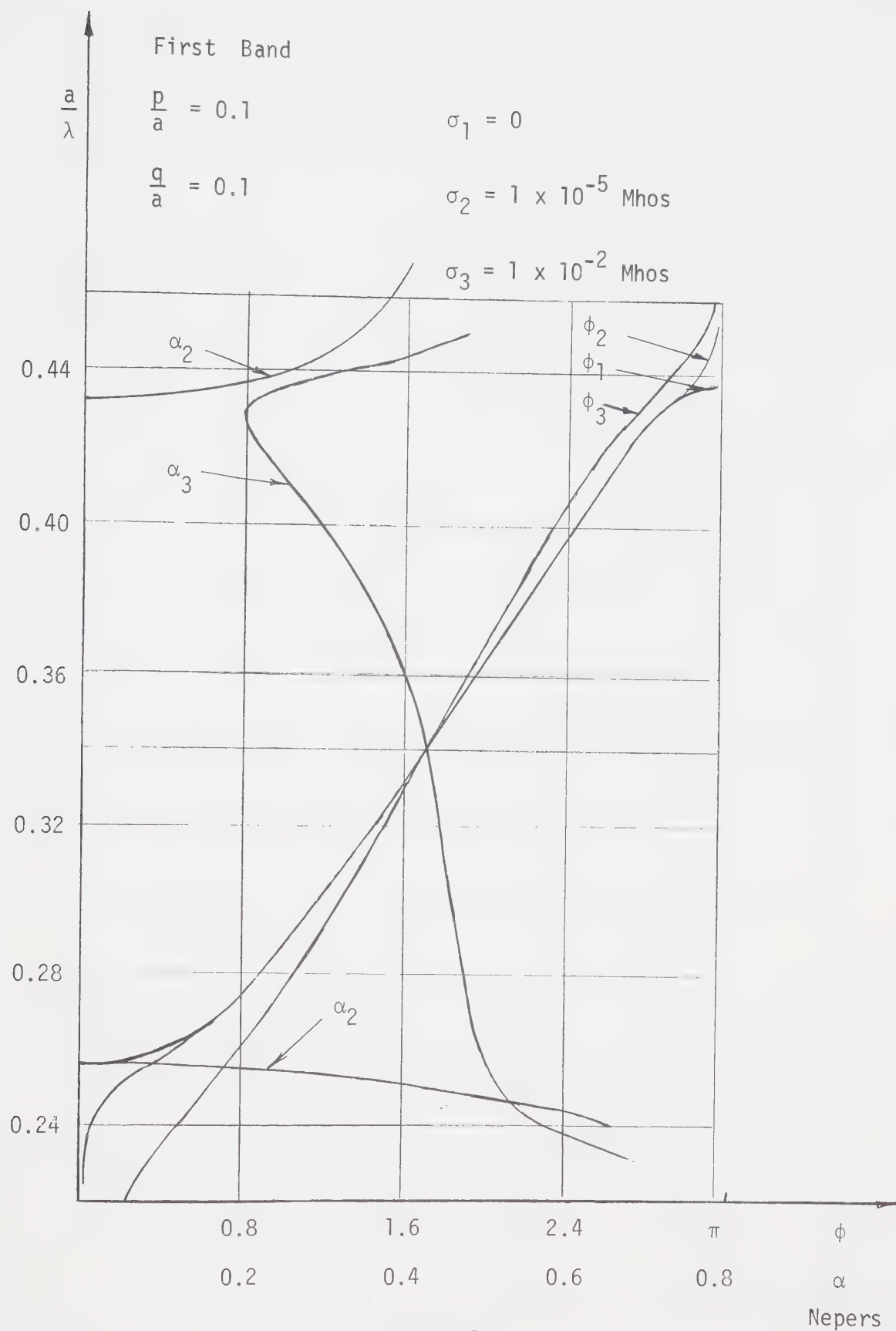


Fig. 2.1



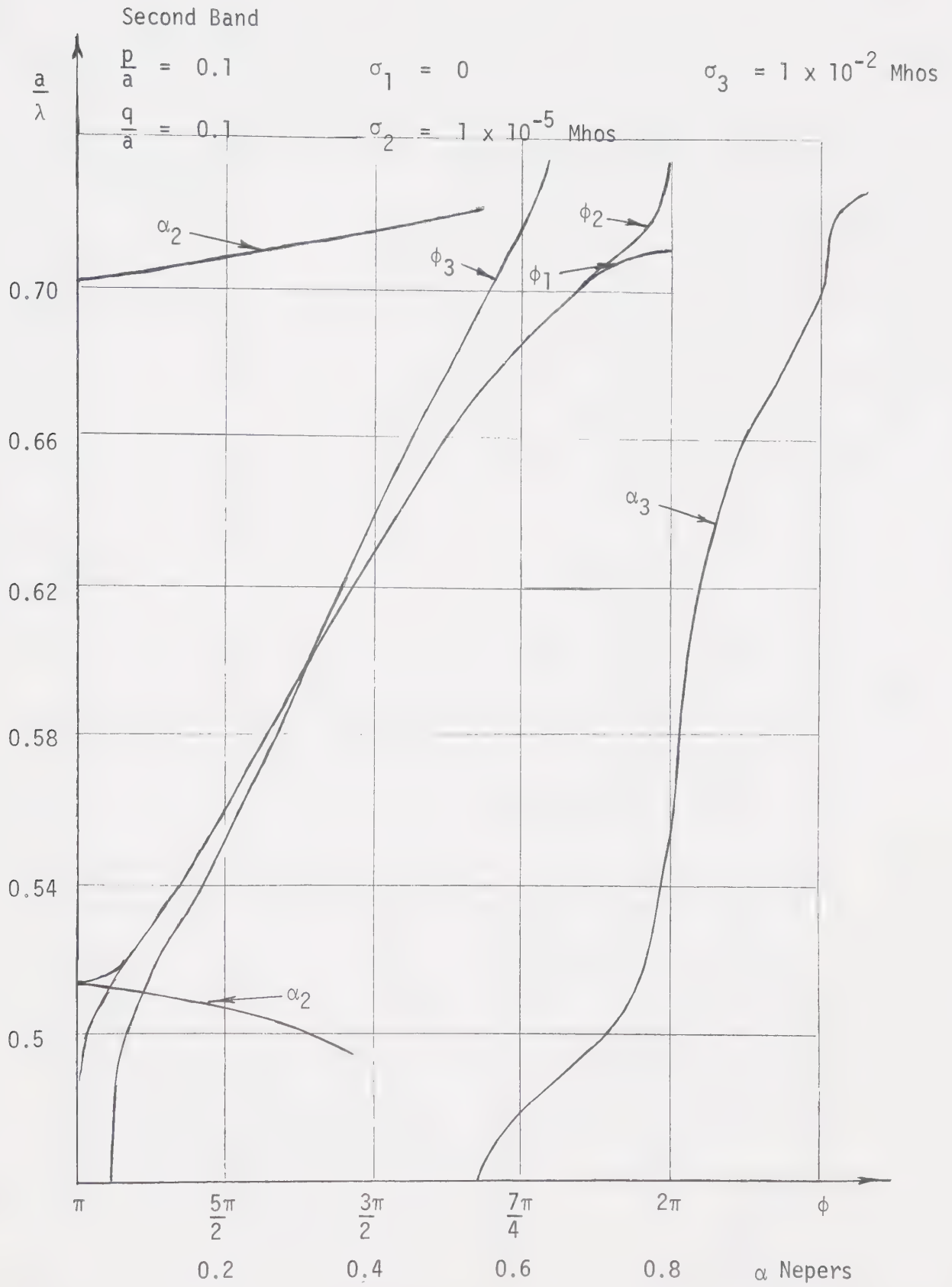


Fig. 2.2





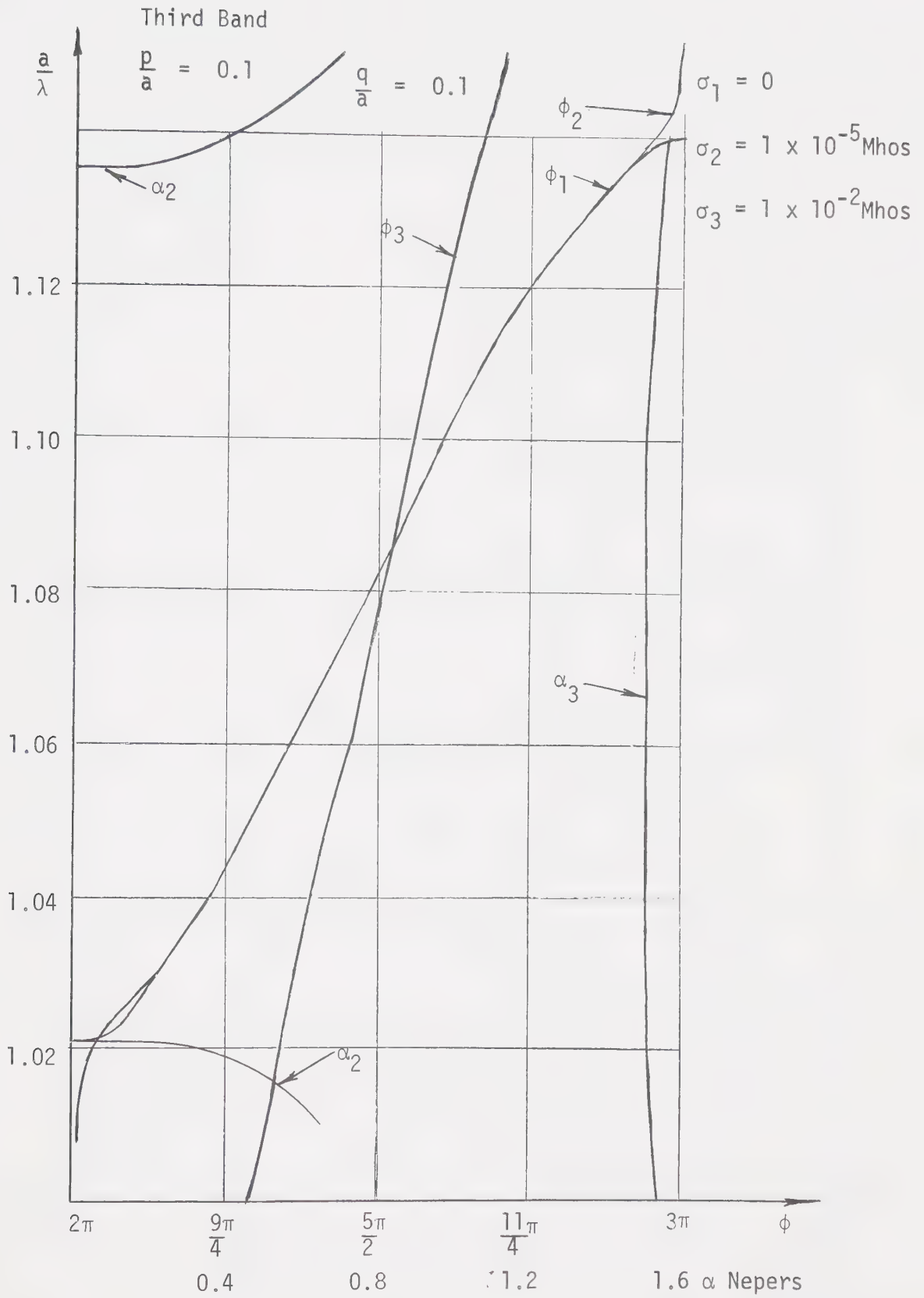


Fig. 2.3



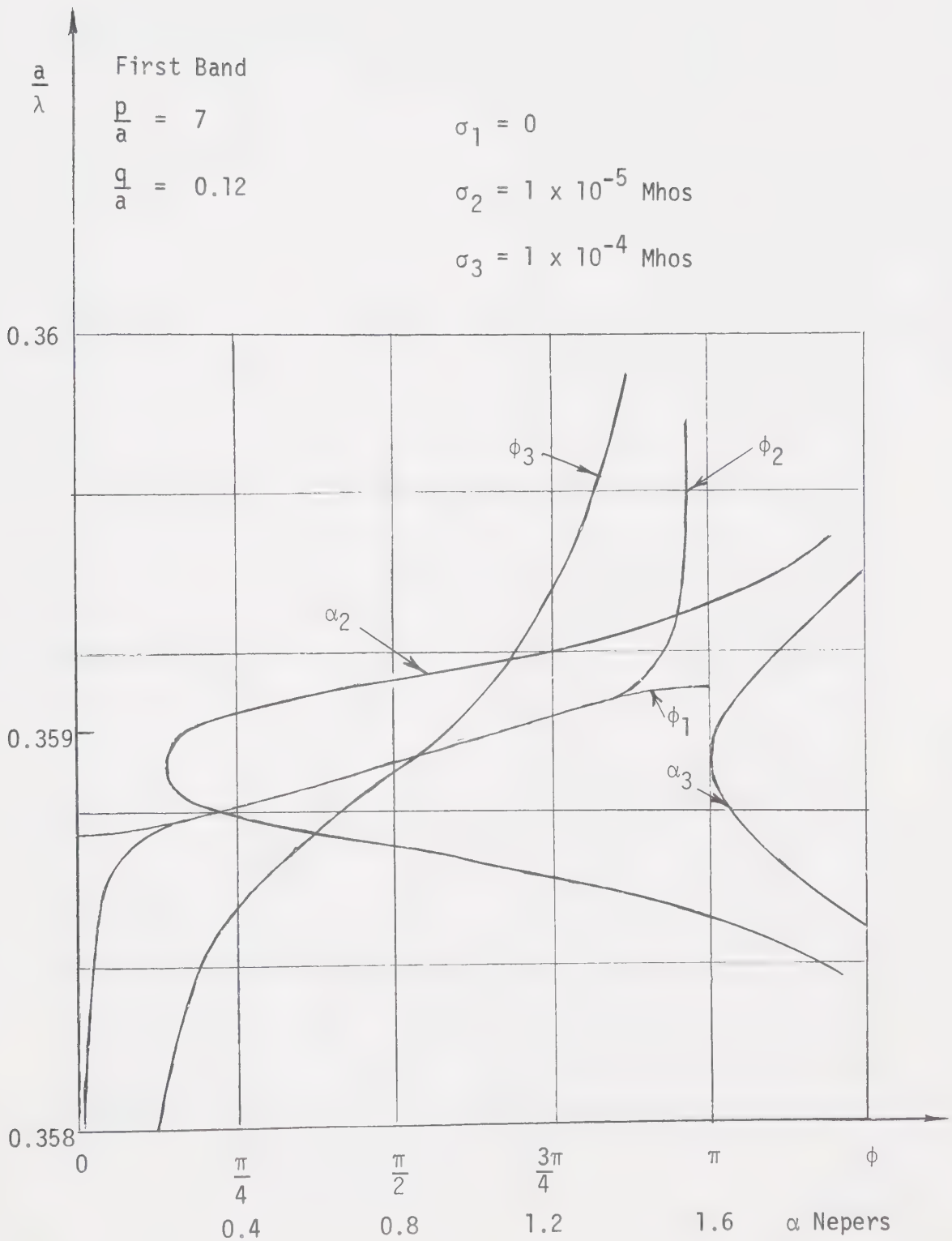


Fig. 2.4



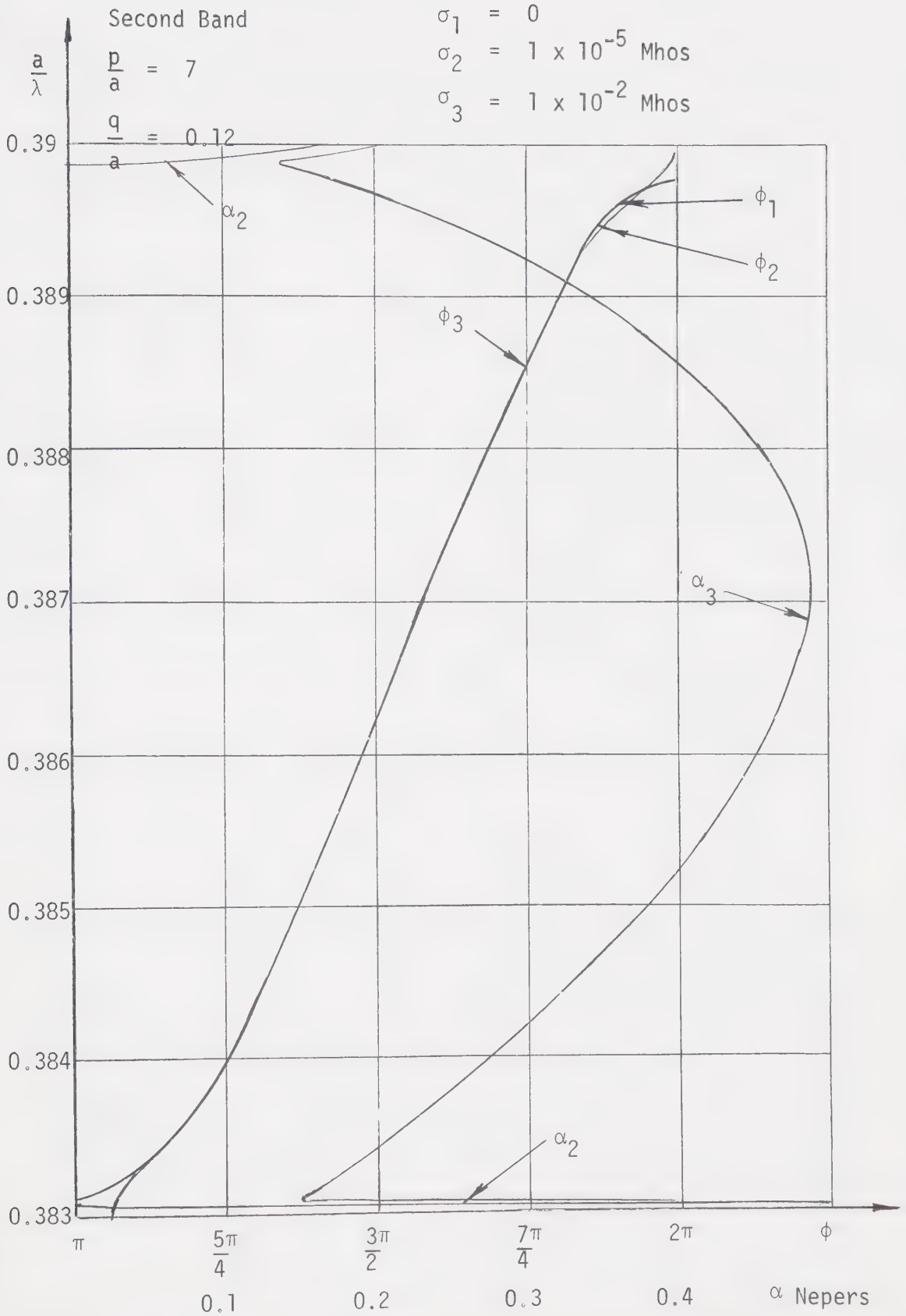


Fig. 2.5



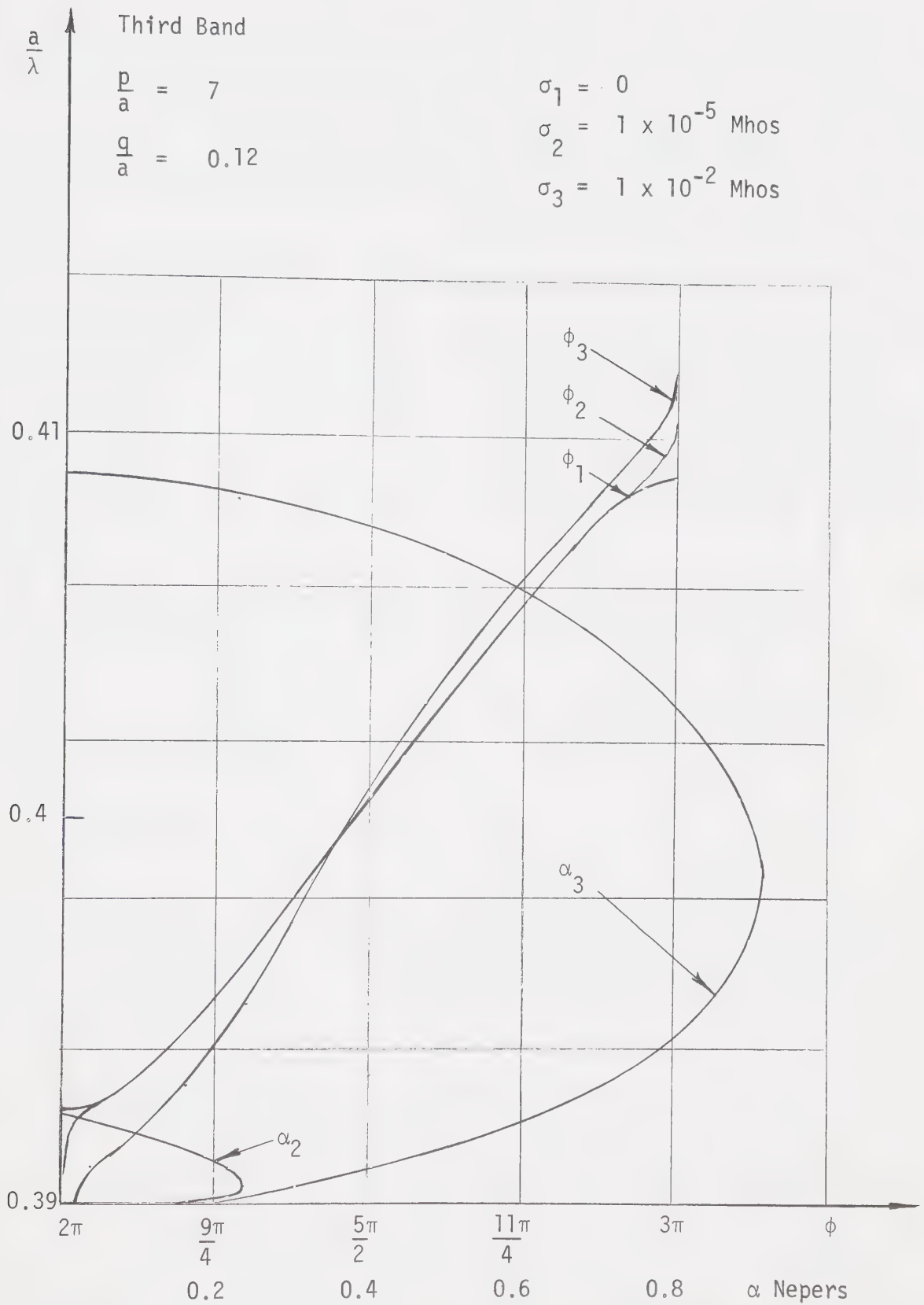


Fig. 2.6





# CHAPTER III

## CALCULATION OF ENERGY STORED POWER LOSS AND Q FACTOR FOR A DIELECTRIC LOADED CAVITY

In this chapter a loaded cavity supporting the  $E_{01}$  mode is considered for the cases where the loading disk is uncoated, and where the disk is coated with a thin layer of metal. It has been assumed that the only effect of the coating is to add to the losses in the cavity. This loss has been found and its effect on the Q values examined.

### 3.1 Calculation of Energy Stored and Power Loss in a Cavity.

When the power loss in the cavity is small the total stored energy is given by either

$$\frac{\mu}{2} \int |H|^2 dv \quad \text{or} \quad \frac{\epsilon}{2} \int |E|^2 dv \quad \dots 3.1$$

where  $|H|$  and  $|E|$  are the amplitude of the magnetic and the electric field intensities at each point. The integral should be taken over the whole volume of the cavity.

a) The symmetric  $E_{01}$  mode ( $E_z$  is symmetric)

The electric field distribution for this mode is such that:

$$E_r = 0 \quad \text{at } z = 0 \quad \text{and at } z = \frac{1}{2} \quad \dots 3.2$$

The field pattern for the  $E_{012}$  mode is shown in figure 3.1



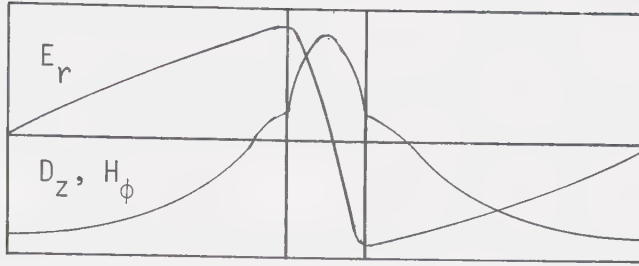


Fig. 3.1 Field Pattern for the symmetric  $E_{012}$  mode.

Substituting  $n = 0$  in equation 1.1 and 1.2 and using the conditions stated by equation 3.2 we get:

$$\begin{aligned}
 E_z &= 2A J_0(\chi r) \cos \beta_d z \\
 E_r &= 2 \frac{\beta_d}{\chi} A J_1(\chi r) \sin \beta_d z \\
 H_\phi &= \frac{2j\omega\epsilon_d}{\chi} A J_1(\chi r) \cos \beta_d z
 \end{aligned}
 \quad \dots 3.3$$

for the air region, and

$$\begin{aligned}
 E_z &= 2M J_0(\chi r) \cos \beta_a \left(z - \frac{1}{2}\right) \\
 E_r &= 2M \frac{\beta_a}{\chi} J_1(\chi r) \sin \beta_a \left(z - \frac{1}{2}\right) \\
 H_\phi &= 2jM \frac{\omega\epsilon_a}{\chi} J_1(\chi r) \cos \beta_a \left(z - \frac{1}{2}\right)
 \end{aligned}
 \quad \dots 3.4$$

for the air region, where

$$M = Ce^{-j\beta_a l}$$

The relation between  $A$  and  $M$  may be evaluated by matching



the transverse field components at the dielectric boundary;

$$E_{rd} = E_{ra} \quad \text{at } z = \frac{q}{2}$$

$$\frac{2\beta_d}{\chi} A J_1(\chi r) \sin \beta_d \frac{q}{2} = \frac{2\beta_a}{\chi} M J_1(\chi r) \sin \beta_a \left( \frac{q}{2} - \frac{1}{2} \right)$$

$$\text{or} \quad \frac{A}{M} = - \frac{\beta_a}{\beta_d} \frac{\sin \theta_1}{\sin \theta_2} \quad \dots 3.5$$

$$= m \quad \dots 3.6$$

### Energy stored.

Equation 3.6 indicates that  $m$  is real i.e.  $A$  is in phase with  $M$ , hence energy stored can be calculated from equation 3.1:

$$U = \frac{\mu}{2} \int |H_{\max}|^2 dv.$$

In the dielectric region

$$U_d = \frac{\mu}{2} \cdot \frac{4 \omega^2 \epsilon_d^2}{\chi^2} A^2 \int_0^a J_1^2(\chi r) \cdot 2\pi r dr \int_{-\frac{q}{2}}^{\frac{q}{2}} \cos^2 \beta_d z dz$$

Substituting for the values of integrals from equations A.2 and A.4 (see appendix).

$$U_d = \frac{\omega^2 \mu \epsilon_d^2}{\chi^2} A^2 \pi a^2 J_1^2(\chi a) \left( q + \frac{\sin 2\theta_2}{\beta_d} \right) \quad \dots 3.7$$

Substituting equation 3.5 in 3.7, we get:

$$U_d = \frac{\omega^2 \mu \epsilon_d^2}{\chi^2} \cdot M^2 \pi a^2 J_1^2(\chi a) m^2 \left( q + \frac{\sin 2\theta_2}{\beta_d} \right) \quad \dots 3.8$$



In the air region:

$$U_a = \frac{\omega^2 \mu \epsilon_a^2}{\chi^2} M^2 \int_0^a J_1^2(\chi r) 2\pi r dr \cdot 2 \int_{\frac{q}{2}}^{\frac{1}{2}} \cos^2 \beta_a (z - \frac{1}{2}) dz \quad \dots 3.9$$

Substituting for the values of the integrals from equations A.3 and A.4, we get:

$$U_a = \frac{\omega^2 \mu \epsilon_a^2}{\chi^2} M^2 \pi a^2 J_1^2(\chi a) \left( p + \frac{\sin 2\theta_1}{\beta_a} \right) \quad \dots 3.10$$

#### Power loss in the metal wall

The power loss in the end wall is given by:

$$W_1 = 2 \frac{R_s}{2} \int_0^a |H_{\phi a}|^2 dz = \frac{1}{2} \chi^2 2\pi r dr$$

$$\therefore W_1 = 4 R_s \frac{\omega^2 \epsilon_a^2}{\chi^2} M^2 \pi a^2 J_1^2(\chi a) \quad \dots 3.11$$

where  $R_s$  is the surface resistivity and is given by:

$$R_s = \sqrt{\frac{\omega \mu}{2\sigma}} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

In the cylindrical walls the losses in the dielectric and the air regions ( $W_2$  and  $W_3$ ) are:

$$W_2 = \frac{R_s}{2} \int_{-\frac{q}{2}}^{\frac{q}{2}} |H_{\phi d}|_{r=a}^2 2\pi a dz$$





$$= 4R_s \omega^2 \epsilon_d^2 A^2 \pi a J_1^2(\chi a) \int_{-\frac{q}{2}}^{\frac{q}{2}} \cos^2 \beta_d z \, dz$$

Substituting from equations 3.5 and A.2 we get:

$$W_2 = 2R_s \frac{\omega^2 \epsilon_d^2}{\chi^2} M^2 \pi a J_1^2(\chi a) m^2 \left( q + \frac{\sin 2\theta_2}{\beta_d} \right) \dots 3.12$$

$$\begin{aligned} W_3 &= \frac{2R_s}{2} \int_{\frac{q}{2}}^{\frac{1}{2}} |H_{\phi a}|_{r=a}^2 2\pi a \, dz \\ &= 8R_s \frac{\omega^2 \epsilon_a^2}{\chi^2} M^2 \pi a J_1^2(\chi a) \int_{\frac{q}{2}}^{\frac{1}{2}} \cos^2 \beta_a \left( z - \frac{1}{2} \right) dz \end{aligned}$$

Substituting from equation A.3 we get:

$$W_3 = 2R_s \frac{\omega^2 \epsilon_a^2}{\chi^2} M^2 \pi a J_1^2(\chi a) \left( p + \frac{\sin 2\theta_1}{\beta_a} \right) \dots 3.13$$

### Power Loss in the dielectric

The power dissipated in the dielectric may be shown<sup>(8)</sup> to be:

$$W_d = \omega \tan \delta \frac{\epsilon_d}{2} \int |E|^2 \, dv \dots 3.14$$

$$\begin{aligned} \therefore W_d &= \omega \tan \delta \frac{\epsilon_d}{2} \int (|E_z|^2 + |E_r|^2) \, dv \\ &= (W_{d1} + W_{d2}) \omega \tan \delta \end{aligned}$$

$$\therefore W_{d1} = \frac{\epsilon_d}{2} \cdot 4A^2 \int_0^a J_0^2(\chi r) \cdot 2\pi r \, dr \int_{-\frac{q}{2}}^{\frac{q}{2}} \cos^2 \beta_d z \, dz$$



and

$$W_{d2} = \frac{\epsilon_d}{2} 4A^2 \frac{\beta_d^2}{\chi^2} \int_0^a J_1^2(\chi r) \cdot 2\pi r dr \int_{-\frac{q}{2}}^{\frac{q}{2}} \sin^2 \beta_d z \, dz$$

Substituting the values of the integrals from equations A.1, A.2, A.4 and A.5, and using equation 3.5 we get:

$$W_d = \omega \tan \delta \, m^2 \epsilon_d \, M^2 \pi a^2 J_1^2(\chi a) \left( q - \frac{\sin 2\theta_2}{\beta_d} + \frac{\beta_d^2}{\chi^2} \left( q + \frac{\sin 2\theta_2}{\beta_d} \right) \right) \quad \dots 3.15$$

b) The antisymmetric  $E_{01}$  mode ( $E_z$  antisymmetric)

The electric field distribution for this mode is such that:

$$E_z = 0 \quad \text{at } z = 0$$

$$E_r = 0 \quad \text{at } z = \frac{1}{2} \quad \dots 3.16$$

The field pattern for the  $E_{011}$  mode is shown in figure 3.2

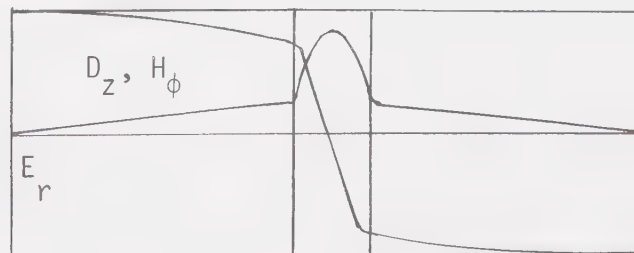


Fig. 3.2 Field Pattern for the antisymmetric  $E_{011}$  mode.



Substituting  $n = 0$  in equation 1.1 and 1.2 and using the conditions stated by equation 3.16, we get:

$$\begin{aligned} E_z &= -2Aj \sin \beta_d z J_0(\chi r) \\ E_r &= \frac{2j\beta_d A}{\chi} \cos \beta_d z J_1(\chi r) \\ H_\phi &= \frac{2\omega\epsilon_d}{\chi} A \sin \beta_d z J_1(\chi r) \end{aligned} \quad \dots 3.17$$

for the dielectric region, and

$$\begin{aligned} E_z &= -2j M' \cos \beta_a \left(z - \frac{1}{2}\right) J_0(\chi r) \\ E_r &= -2j M' \frac{\beta_a}{\chi} \sin \beta_a \left(z - \frac{1}{2}\right) J_1(\chi r) \\ H_\phi &= 2M' \frac{\omega\epsilon_a}{\chi} \cos \beta_a \left(z - \frac{1}{2}\right) J_1(\chi r) \end{aligned} \quad \dots 3.18$$

where  $M' = jCe^{-j\beta_a l}$

The value of  $\frac{A}{M'}$  may be obtained by matching the transverse field components at the boundaries:

$$E_{rd} = E_{ra} \quad \text{at} \quad z = \frac{g}{2}$$

from which we get:

$$\begin{aligned} \frac{A}{M'} &= \frac{\beta_a}{\beta_d} \frac{\sin \theta_1}{\cos \theta_2} \\ &= m' \end{aligned} \quad \dots 3.19$$



The same procedure as for the symmetric  $E_{01}$  mode can be used in this case to find expressions for the energy stored and the power loss. These are found to be:

$$U_d = \frac{\omega^2 \mu \epsilon_d^2}{\chi^2} M'^2 \pi a^2 J_1^2 (\chi a) m'^2 \left( q - \frac{\sin 2\theta_2}{\beta_d} \right) \dots 3.20$$

$$U_a = \frac{\omega^2 \mu \epsilon_a^2}{\chi^2} M'^2 \pi a^2 J_1^2 (\chi a) \left( p + \frac{\sin 2\theta_1}{\beta_a} \right) \dots 3.21$$

$$W_1 = 4R_s \frac{\omega^2 \epsilon_a^2}{\chi^2} M'^2 \pi a^2 J_1^2 (\chi a) \dots 3.22$$

$$W_2 = 2R_s \frac{\omega^2 \epsilon_d^2}{\chi^2} M'^2 \pi a^2 J_1^2 (\chi a) m'^2 \left( q - \frac{\sin 2\theta_2}{\beta_d} \right) \dots 3.23$$

$$W_3 = 2R_s \frac{\omega^2 \epsilon_a^2}{\chi^2} M'^2 \pi a^2 J_1^2 (\chi a) \left( p + \frac{\sin 2\theta_1}{\beta_a} \right) \dots 3.24$$

$$W_d = \omega \tan \delta m'^2 \epsilon_d M'^2 a^2 J_1^2 (\chi a) \left( q + \frac{\sin 2\theta_2}{\beta_d} + \frac{\beta_d^2 d}{\chi^2} \left( q - \frac{\sin 2\theta_2}{\beta_d} \right) \right) \dots 3.25$$

### 3.2 Effect of Coating on the Expressions for Energy Stored and Power Loss.

It was explained in section 2.2 that a very thin coating has to be used in order that propagation takes place. For this case there is a negligible change in the field pattern. Thus expressions for energy stored and power loss derived earlier still hold, but an additional loss term arises due to the coating.





The loss due to the coating on one side is given by:

$$\begin{aligned}
 W_c &= \frac{R_1}{2} \int_0^a |J|^2 2\pi r dr \\
 &= \frac{\sigma_1'}{2} \int_0^a |E_{rd}|^2_{z=\frac{a}{2}} 2\pi r dr
 \end{aligned}$$

where  $\sigma_1'$  is the conductance of the layer.

The loss due to the coating on both sides is given by twice the above value.

For symmetrical mode, therefore,

$$W_4 = \frac{4\sigma_1' \beta_d^2}{\chi^2} M^2 \pi a^2 J_1^2(\chi a) m^2 \sin^2 \theta_2 \quad \dots 3.26$$

Similarly for the antisymmetric  $E_{01}$  mode

$$W_4 = \frac{4\sigma_1' \beta_d^2}{\chi^2} M'^2 \pi a^2 J_1^2(\chi a) m'^2 \cos^2 \theta_2 \quad \dots 3.27$$

It is interesting to note that, for the symmetrical case, the transverse field component  $E_r$  is a function of  $\sin \beta_d z$ , thus the loss is equal to zero if  $\theta_2 = 0, \pi, \dots$ . For the antisymmetrical case  $E_r$  is a function of  $\cos \beta_d z$ , and the loss is equal to zero if

$$\theta_2 = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

### 3.3 Expression for Q Values of a Cavity.

The unloaded Q factor of a cavity is defined as:



$$Q = \frac{\omega \times \text{Energy stored}}{\text{power dissipated}} \quad \dots 3.28$$

It follows that,

$$\frac{1}{Q} = \frac{\Sigma \text{ Losses}}{\omega \times \text{Energy stored}}$$

so that

$$\frac{1}{Q} = \frac{1}{Q_m} + \frac{1}{Q_d} + \frac{1}{Q_c}$$

Where  $Q_m$ ,  $Q_d$ ,  $Q_c$  are the Q factors taking only metal, dielectric and coating loss into consideration.

For the evaluation of  $Q_d$  it may be noted that (9),

$$Q_d = \frac{1}{\tan \delta} \left( 1 + \frac{\frac{\epsilon_0}{2} \int_{\text{air}} |E|^2 dv}{\frac{\epsilon_d}{2} \int_{\text{die}} |E|^2 dv} \right) \quad \dots 3.29$$

The electric energy stored in the dielectric may be obtained from equations 3.15 and 3.25, and the energy stored in the electric field in the air region is given by:

$$\begin{aligned} U_{Ea} &= \frac{\epsilon_a}{2} \int_{\text{air}} |E|^2 dv \\ &= \frac{\epsilon_a}{2} \int_{\text{air}} (|E_r|^2 + |E_z|^2) dv \end{aligned}$$

For the symmetric  $E_{01}$  mode:

$$U_{Ea} = \epsilon_a J_1^2 (\chi a) a^2 M^2 \pi \left( p + \frac{\sin 2\theta_1}{\beta_a} + \frac{\beta_a^2}{\chi^2} \left( p - \frac{\sin 2\theta_1}{\beta_a} \right) \right) \quad \dots 3.30$$



Similarly for the antisymmetric  $E_{01}$  mode

$$U_{Ea} = \epsilon_a J_1^2(\chi a) a^2 M'^2 \pi \left( p + \frac{\sin 2\theta_1}{\beta_a} + \frac{\beta_a^2}{\chi^2} \left( p - \frac{\sin 2\theta_1}{\beta_a} \right) \right) \quad \dots 3.31$$

Expressions for  $Q_m$ ,  $Q_d$ , and  $Q_c$  were derived, using equations in section 3.1, 3.2 and the above two equations and are found to be

$$Q_m = \frac{\omega a \mu}{2R_s} \left\{ \frac{2a}{\epsilon_r^2 m^2 \left( q + \frac{\sin 2\theta_2}{\beta_d} \right) + \frac{\sin 2\theta_1}{\beta_a} + p} \right\}^{-1} \quad \dots 3.32$$

$$Q_d = \frac{1}{\tan \delta} \left\{ \frac{\left( p + \frac{\sin 2\theta_1}{\beta_a} \right) + \frac{\beta_a^2}{\chi^2} \left( p - \frac{\sin 2\theta_1}{\beta_a} \right)}{q + \frac{\sin 2\theta_2}{\beta_d} + \frac{\beta_d^2}{\chi^2} \left( q - \frac{\sin 2\theta_2}{\beta_d} \right)} \times \frac{1}{m^2 \epsilon_r} + 1 \right\} \quad \dots 3.33$$

$$Q_c = \frac{\omega \mu \epsilon_a^3 \epsilon_r^2 m^2 \left( q + \frac{\sin 2\theta_2}{\beta_d} \right) + p + \frac{\sin 2\theta_1}{\beta_a}}{4\sigma_1' \beta_a^2 \sin^2 2\theta_1} \quad \dots 3.34$$

for the symmetric mode, and

$$Q_m = \frac{\omega a \mu}{2R_s} \left\{ \frac{2a}{\epsilon_r^2 m'^2 \left( q - \frac{\sin 2\theta_2}{\beta_d} \right) + p + \frac{\sin 2\theta_1}{\beta_a}} \right\}^{-1} \quad \dots 3.35$$



$$Q_d = \frac{1}{\tan \delta} \left\{ \frac{p + \frac{\sin 2\theta_1}{\beta_a} + \frac{\beta_a^2}{\chi^2} (p - \frac{\sin 2\theta_1}{\beta_a})}{q - \frac{\sin 2\theta_2}{\beta_a} + \frac{\beta_d^2}{\chi^2} (q + \frac{\sin 2\theta_2}{\beta_d}) m'^2 \epsilon_r} + 1 \right\}$$

$$Q_c = \frac{\omega_\mu^2 \epsilon_a^2}{4\sigma_1} \left( \frac{\epsilon_r m'^2 (q - \frac{\sin 2\theta_2}{\beta_d}) + p + \frac{\sin 2\theta_1}{\beta_a}}{\beta_a^2 \sin^2 2\theta_1} \right) \quad \dots\dots 3.36$$

\dots\dots 3.37

for the antisymmetric mode.



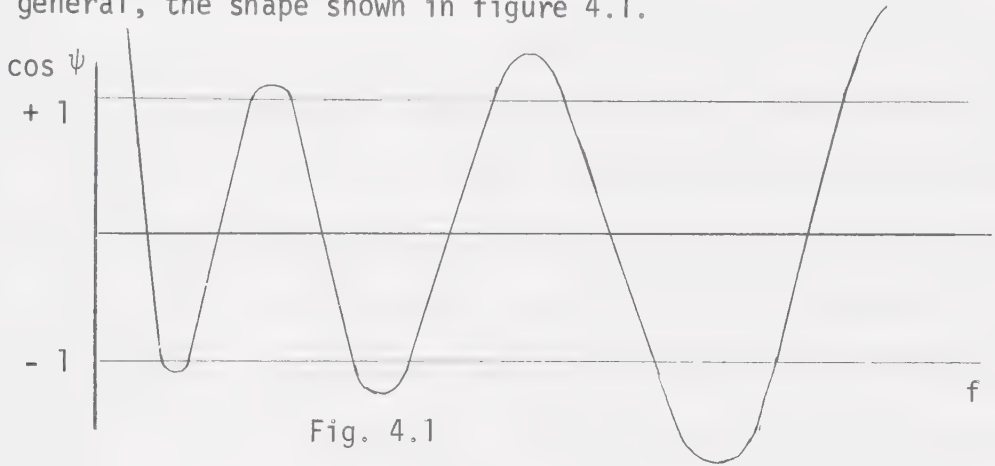


## CHAPTER IV

### DESCRIPTION OF CAVITY RESONATOR AND EXPERIMENTAL SET-UP

#### 4.1 Cavity Resonator as a Part of Loaded Waveguide.

The phase shift per section ( $\psi$ ) may be plotted against frequency (equation 1.5). This graph for any structure is found to have, in general, the shape shown in figure 4.1.



This graph shows that a periodically loaded structure acts like a band pass filter. The pass bands occur when  $|\cos \psi| \leq 1$ . It was noticed that both the pass bands and the stop bands become wider as the frequency increases.

Two shorting planes may be placed in the middle of the air region one period apart to form a resonant cavity. Resonant frequencies for this cavity occur when  $\psi$  is an integral multiple of  $\pi$ . Since the tangential electric field,  $E_r$  for the  $E_{01}$  mode, at the shorting planes



must be zero, another two conditions can be found from equation 1.2.

For  $E_r = 0$  at  $z = \frac{1}{2}$  these conditions are: (10,11)

$$\tan \theta_1 \tan \theta_2 = \frac{Z_d}{Z_a} \quad \text{for } \psi = n\pi \text{ (n is an odd integer)}$$

$$\tan \theta_1 \cot \theta_2 = \frac{-Z_d}{Z_a} \quad \text{(n is an even integer).}$$

....4.1

#### 4.2 Design and Construction of Resonator.

The cavity dimensions are governed by a number of factors. The diameter of the resonator was chosen to support the  $E_{01}$  mode at J band (5.3 - 8.2 GHz). The diameter should be smaller than the cut-off diameter of the  $E_{02}$  mode. The length of the air region governs the number of resonant frequencies within a certain band of frequency; the longer this region is the larger the number of resonant frequencies. A probe type coupling at the centre of the end walls of the cavity was used to avoid the excitation of the H modes.

A cylindrical cavity suitable for the investigation of coating is shown in figure 4.5. It has been found convenient to assemble the resonator from two parts. A silvered copper ring is shrunk onto the disk and the ring is then clamped at the junction of these two parts. The inside of the cavity was silver plated but it was not polished. The coupling holes at the centre of the cavity end plates have to be very small so that the effect on field pattern and thus the resonant frequency



is negligible.

### 4.3 Apparatus Used.

The set up for measuring the  $Q$  of the cavity and the resonant frequency is described with reference to the block diagram shown in figure 4.7. Klystron  $K_1$  is frequency modulated using a saw tooth generator so that the  $Q$  curve of the cavity under test may be displayed on the Oscilloscope. The same signal is used to drive the time base of the Oscilloscope. A ferrite isolator  $F$  is used to isolate the Klystron  $K_1$  from the rest of the system. A precision, calibrated, well matched attenuator is used to indicate the 3 db point on the  $Q$  curve. The tuning stubs  $S_1$  and  $S_2$  serve to match the input and output impedances. An Oscilloscope which has a high gain amplifier is used to amplify the detected signal. A magic tee  $T$  is used for mixing a fraction of the modulated frequency from klystron  $K_1$  with the output of the microwave generator  $K_2$ . A second mixer  $M$  is used to mix the detected output from mixer  $T$  with the output of a radio frequency signal generator  $Z$ . The internal circuit of the mixer  $M$  is connected as shown in figure 4.6. This mixer uses an integrated circuit element Motorola type MC 1550. The resistance  $R$  was adjusted such that the currents in terminals 2 and 3 were equal. For the circuit shown it was found to be  $1.3 \text{ K } \Omega$ . The gain of the mixer can be adjusted by changing the value of the resistance connected between output terminals 9 and 6.



#### 4.4 Procedure for Measurement of Q Values.

The procedure for the Q measurement with reference to the response curve on the Oscilloscope is as follows:

The modulation of the klystron  $K_1$  is adjusted to give an adequate frequency coverage. Input and output tuning stubs and the insertion of the probes are adjusted to give the best response on the Oscilloscope. It is important to check that the coupling is very loose. This is easily done by noting the change in resonant frequency and Q of the cavity when probes are slightly moved in and out of the cavity. For loose coupling no change in Q or frequency is observed. The frequency is measured directly by an absorption type wavemeter (W). A fraction of the modulated signal is applied via a directional coupler (D), to the magic tee where it is mixed with the constant frequency from the second klystron. The detected output which contains the sum and the difference of these two microwave frequencies is fed to the second mixer. The output is connected to channel 2 of the Oscilloscope. When the klystron  $K_1$  and the microwave generator  $K_2$  are in tune a large pip appears. To get accurate results this pip must be partially suppressed. This was done by the filter circuit at the input of the mixer (as shown in figure 4.6). Pips also appear at the instant when the output frequency from the mixer T is equal to the signal generator frequency or its multiples; these lie on either side of the tune position. The amplitude of the pips decreases as the order of the pips increases as shown in Figure 4.3.





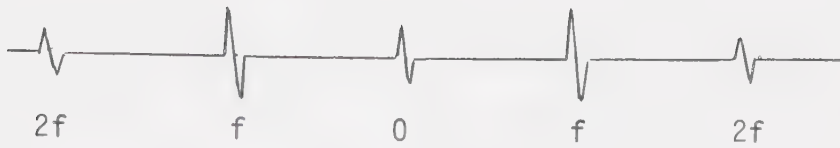


Fig. 4.3

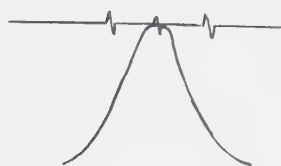
The determination of the fractional bandwidth is carried out as follows:-

- 1) The relative position of the response curve and the pips are adjusted on the Oscilloscope so that the centre pip is coincident with the peak of the resonant curve as shown in Figure 4.4a. This can be done by changing the frequency of the microwave generator  $K_2$ .
- 2) The precision attenuator A is used to increase the input power to the cavity by 3 db. The trace of channel (2) is now at the 3 db. level. The frequency of the r.f generator  $Z$  may be adjusted so that the side pips coincide with the half power points as shown in Figure 4.4b. The precision attenuator is used to eliminate the uncertainty of the square law of the crystal.

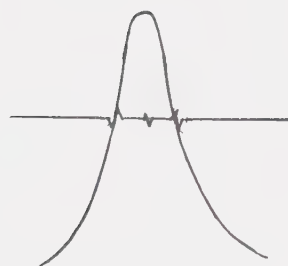
$$\text{Then } Q = \frac{f_r}{2f}$$

where  $f_r$  is the resonant frequency of the cavity and  $f$  is the frequency of the r.f. generator G.





(a)



(b)

Fig. 4.4



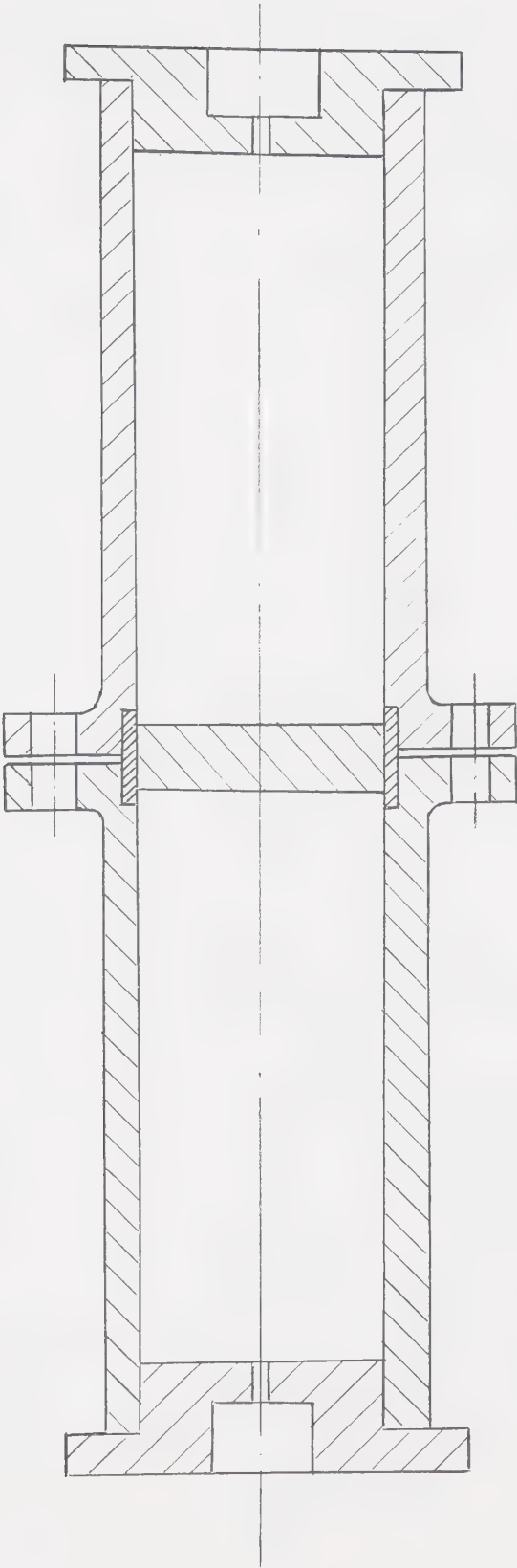


Fig. 4.5 Cavity Resonator



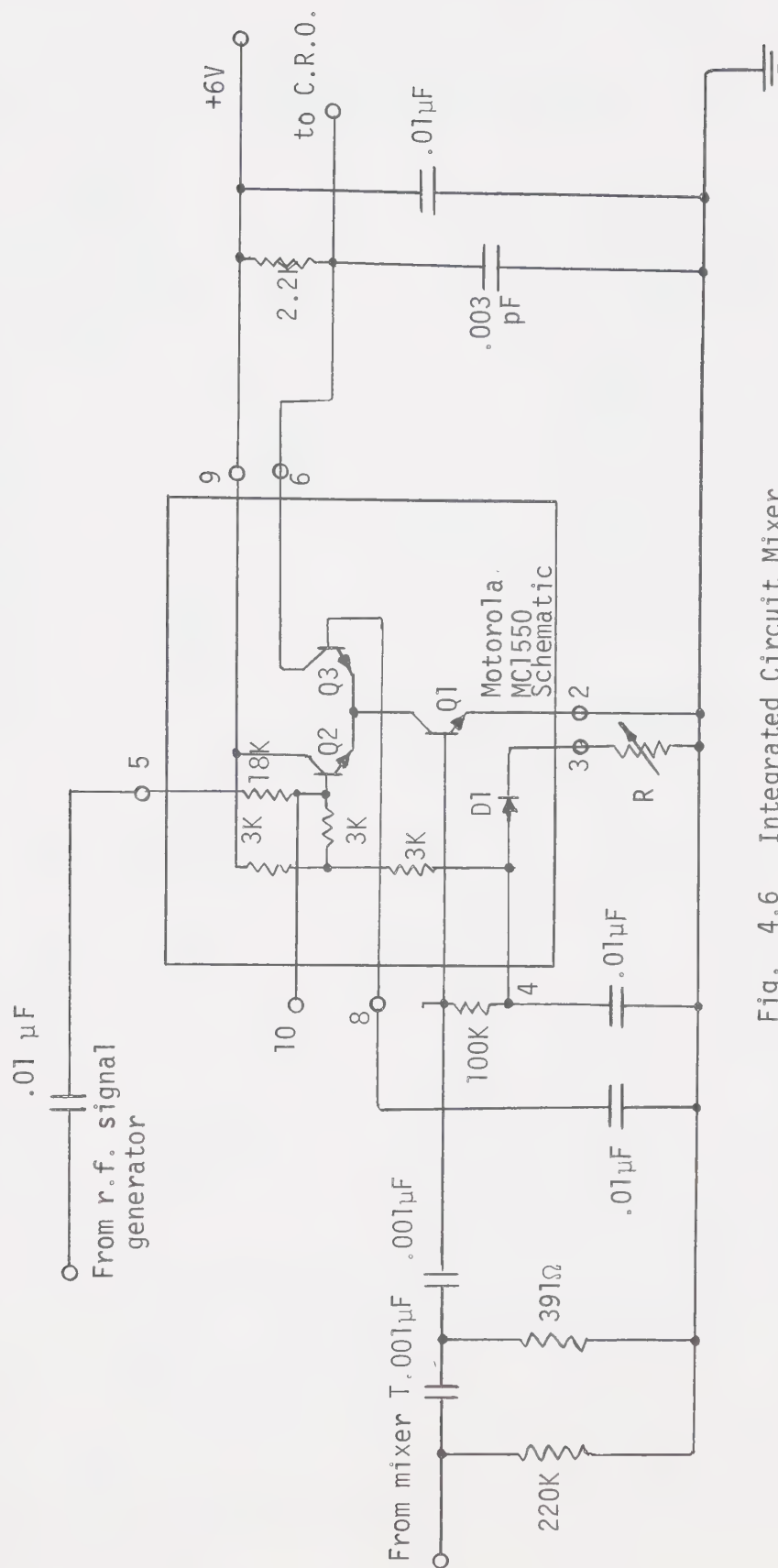


Fig. 4.6 Integrated Circuit Mixer









### KEY TO BLOCK DIAGRAM

|            |                                                           |
|------------|-----------------------------------------------------------|
| A          | - Precision well matched attenuator                       |
| $C_1, C_2$ | - Coupling probes                                         |
| D          | - Directional coupler                                     |
| F          | - Ferrite isolator                                        |
| G          | - Matched waveguide to coaxial termination.               |
| $K_1$      | - Frequency modulated klystron oscillator (J band)        |
| $K_2$      | - Microwave signal generator                              |
| M          | - Integrated circuit mixer                                |
| O          | - Saw tooth generator                                     |
| P          | - Klystron power supply                                   |
| R          | - Resonant cavity under test                              |
| $S_1$      | - Double tuning stub                                      |
| $S_2$      | - Tuning stub and crystal detector                        |
| T          | - Magic-tee used as a mixer for two microwave frequencies |
| V          | - Variable attenuator                                     |
| W          | - Absorbtion wavemeter                                    |
| Z          | - Radio frequency signal generator                        |
| C.R.O.     | - Cathode Ray Oscilloscope                                |



## CHAPTER V

### EXPERIMENTAL RESULTS

#### 5.1 Cavity Dimension and Resonant Frequencies

The cavity used in the measurement is of the type shown in Figure 4.5. In order to obtain several resonant frequencies within the J band (5.3 - 8.2 GHz), several periodic structures which have cut-off frequency for the  $E_{01}$  mode slightly above the lower limit of the J band, were analysed using a computer program. The effect of the metal film on the Q's of the different resonances was also calculated. From the calculations  $\frac{p}{a}$  and  $\frac{q}{a}$  were chosen equal to 7 and .12 respectively, also the radius (a) was chosen equal to 2 cm; this gives 7 resonances that can be used.

The cavity which uses one section of this structure has the dimensions:

$$a = 2 \text{ cm}, \quad l = 14.24 \text{ cm}, \quad q = 0.24 \text{ cm}.$$

By the aid of the computer, the values of  $\frac{a}{\lambda}$  which correspond to  $\cos \psi = \pm 1$  were found. The points which satisfy equation 4.1 are the resonant frequencies for this cavity. These data and the  $\cos \psi$  vs  $\frac{a}{\lambda}$  plot for this structure is shown in Table 5.1 and Figure 5.1 respectively. The asterisks, in Table 5.1 denote points which do not satisfy equation 4.1, i.e. no resonance occurs at these points.



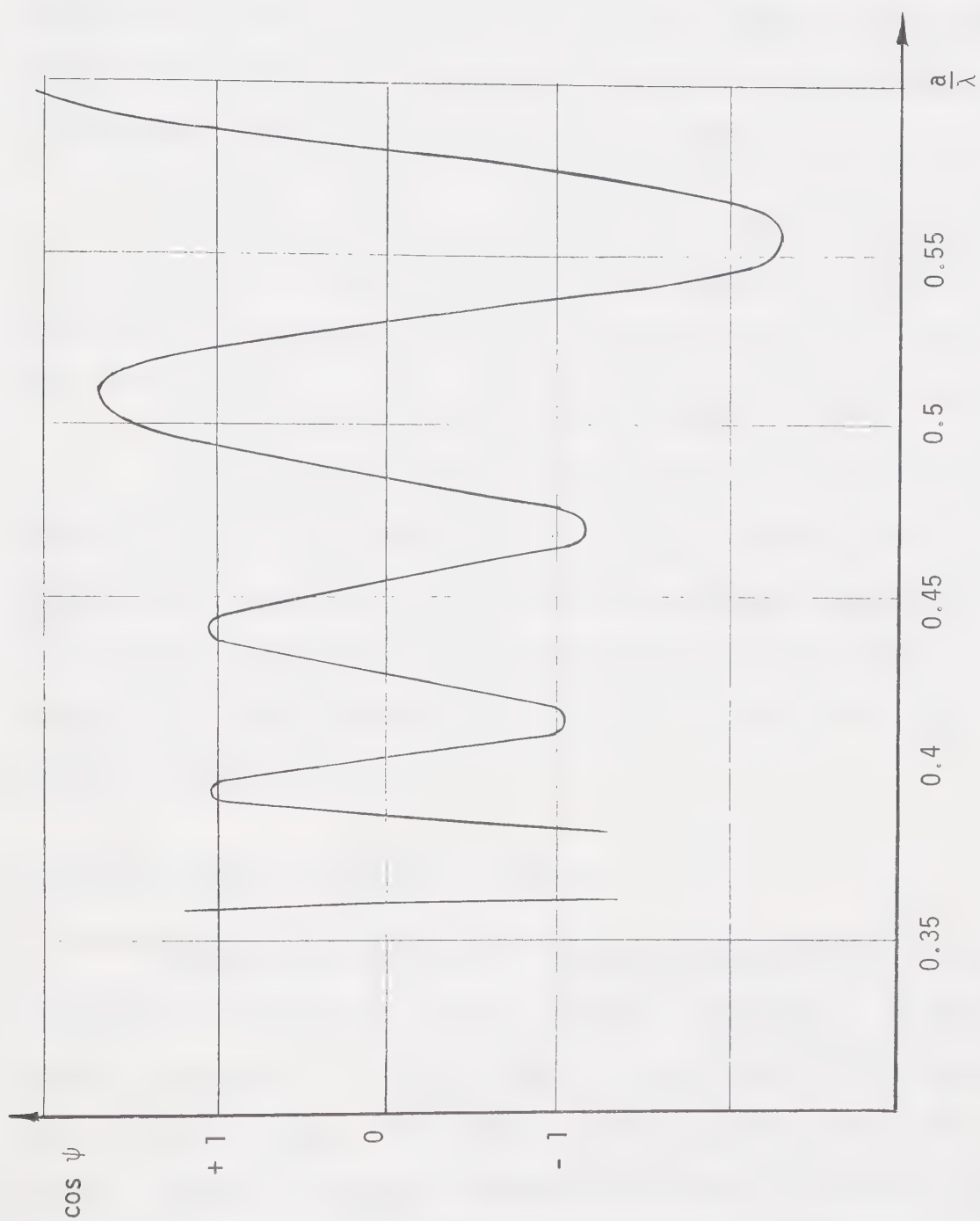


Fig. 5.1





The agreement between theoretical and experimental values of frequencies is limited by the tolerances in machining and the accuracy of the  $\epsilon_r$  value used in the calculation. However the measured and the computed values were fairly close. In Table 5.1 these values are compared. Five of these frequencies lying between 5.6 - 7.3 GHz were considered for Q measurement. The first of these is the  $E_{011}$  mode.

Axial and transverse electric field distribution for these five modes are shown in Figures 5.2 - 5.6. The corresponding values of computed resonant frequencies and the transverse electric field at the surface of the dielectric, normalized to maximum, are noted beside each figure.

It is readily seen that for some of these modes, the transverse electric field at the dielectric surface ( $E_{rd}$ ) is high and thus the loss  $(\frac{1}{2} \int \sigma_1 E_{rd}^2)$  is high too. If the loss is high enough, these modes may be completely suppressed. For other modes  $E_{rd}$  is small. It follows that the Q values for a given thickness of metal, will be slightly or highly reduced depending on the field pattern.

## 5.2 Loss Tangent of the Dielectric Disk

The measured Q values for a loaded cavity were used to calculate  $\tan \delta$  of the dielectric disk, by using equation 3.33 or 3.36. As the energy stored in the dielectric disk was found to be much smaller than the energy stored in the air region, this method is not an accurate one for this purpose. However the values of  $\tan \delta$  obtained compare well with previously measured values for titanium dioxide ceramic ( $\tan \delta = .0003$  Luthra<sup>(9)</sup>).



$$\begin{aligned}\psi &= \pi \\ f_r &= 5.7 \text{ GHz} \\ \frac{E_r}{E_{r \text{ max.}}} &= \cos \theta_2 \\ &= 0.1607\end{aligned}$$

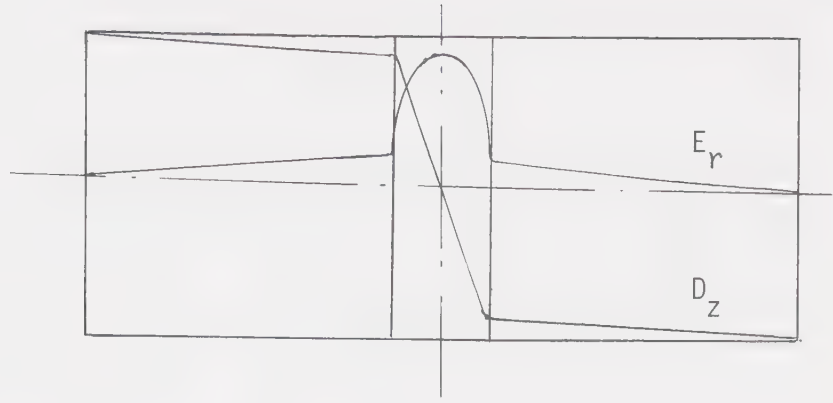


Fig. 5.2 Field pattern for the  $E_{011}$  mode.

$$\begin{aligned}\psi &= 2\pi \\ f_r &= 5.85 \text{ GHz} \\ \frac{E_r}{E_{r \text{ max.}}} &= \sin \theta_2 \\ &= 0.9903\end{aligned}$$

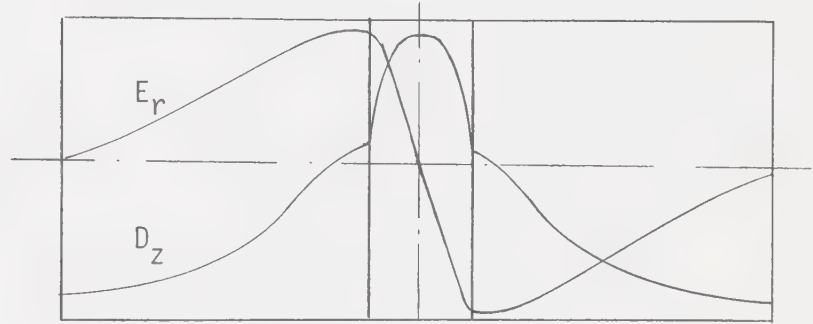


Fig. 5.3 Field pattern for the  $E_{012}$  mode.

$$\begin{aligned}\psi &= 3\pi \\ f_r &= 6.12 \text{ GHz} \\ \frac{E_r}{E_{r \text{ max.}}} &= \cos \theta_2 \\ &= 0.0872\end{aligned}$$

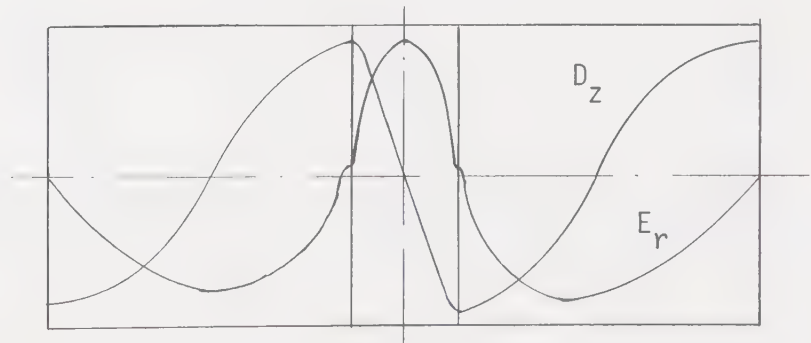


Fig. 5.4 Field pattern for the  $E_{013}$  mode.



$$\begin{aligned}\psi &= 4\pi \\ f_r &= 6.56 \text{ GHz} \\ \frac{E_r}{E_{r \max.}} &= \sin \theta_2 \\ &= 0.999\end{aligned}$$

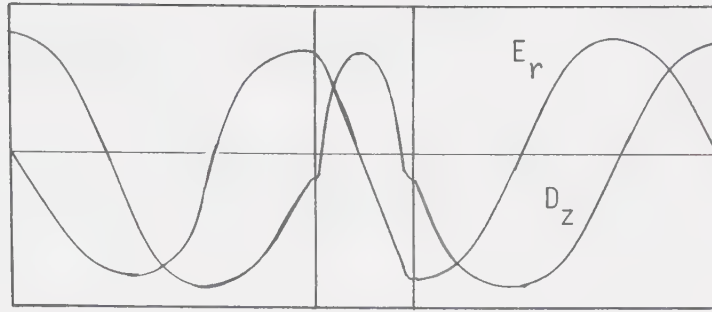


Fig. 5.5 Field pattern for the  $E_{014}$  mode.

$$\begin{aligned}\psi &= 5\pi \\ f_r &= 7.02 \text{ GHz} \\ \frac{E_r}{E_{r \max.}} &= \cos \theta_2 \\ &= -0.165\end{aligned}$$

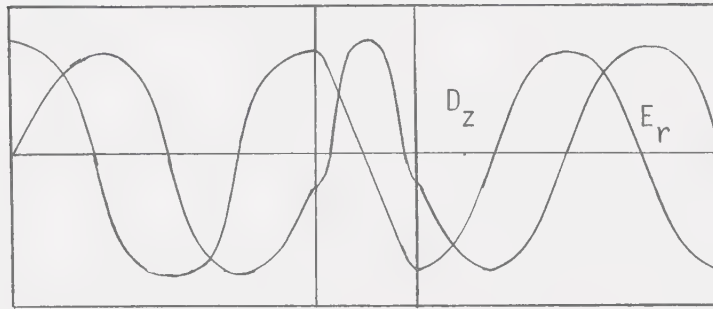


Fig. 5.6 Field pattern for the  $E_{015}$  mode.



These results are shown in Table 5.2.

### 5.3 Effective Resistance of the Coating.

To measure the resistance of the coating 7 similar disks were prepared. One of them was left uncoated, while the other six were coated as follows:

- (a) 200  $\text{\AA}$  of Platinum on both sides.
- (b) 100  $\text{\AA}$  of Aluminum on both sides.
- (c) 200  $\text{\AA}$  of Aluminum on both sides.
- (d) Two disks coated on both sides with carbon (aquadag, thickness unknown).
- (e) 200  $\text{\AA}$  of Aluminum on one side.

The Platinum film was prepared by an Evaporation method while the Aluminum film was prepared by the triode sputtering method; the latter gives a more even coating.

The Carbon coating was prepared by spraying aquadag solution on the surface of the disk.

The  $Q$  of the cavity was measured for the five different modes for each disk. These results are shown in Tables 5.3-5.7. For some modes the losses are very high and  $Q$  values could not be measured by this method.

From Tables 5.3-5.7 the values of the conductance of the coating were calculated by using equation 3.34 or 3.37, the results are





| $\cos\psi$ | $\psi$ | $\frac{a}{\lambda}$ | Resonance frequency | Measured $f_r$ |
|------------|--------|---------------------|---------------------|----------------|
| -1         | $\pi$  | .383                | 5.7 GHz             | 5.739          |
| +1         | $2\pi$ | .39*                |                     |                |
| +1         | $2\pi$ | .392                | 5.85                | 5.901          |
| -1         | $3\pi$ | .409                | 6.12                | 6.13           |
| -1         | $3\pi$ | .412*               |                     |                |
| +1         | $4\pi$ | .437                | 6.56                | 6.72           |
| +1         | $4\pi$ | .4385*              |                     |                |
| -1         | $5\pi$ | .465*               |                     |                |
| -1         | $5\pi$ | .474                | 7.02                | 7.142          |
| +1         | $6\pi$ | .494                | 7.44                |                |
| +1         | $6\pi$ | .5218*              |                     |                |

\* see text page 50

Table 5.1

| Mode      | $Q_m$ | $Q$  | $Q_d \times 10^4$ | $\tan \delta$ | $U_a/U_d$ |
|-----------|-------|------|-------------------|---------------|-----------|
| $E_{011}$ | 8070  | 7500 | 14.3              | .001          | 114       |
| $E_{012}$ | 5300  | 4370 | 2.5               | .000549       | 13.7      |
| $E_{013}$ | 6103  | 5850 | 1.25              | .00045        | 56.4      |
| $E_{014}$ | 6538  | 3580 | 0.875             | .00043        | 2.77      |
| $E_{015}$ | 8360  | 7950 | 14.3              | .000435       | 62        |

Table 5.2



shown in Table 5.8.

Satisfactory results for the disks coated on one side could not be obtained. The half-power bandwidth measured for the odd modes depended on whether the coating was in the input or the output side of the cavity. The curves were not symmetric and for the  $E_{013}$  mode two peaks were observed. For the even modes the losses are high and a Q curve could not be obtained.

It is noticed that these results are not consistent for measurements on the same coating using different modes. Only the  $E_{012}$  and  $E_{014}$  modes give approximately the same values. The wide difference in the results for the other modes can be explained by considering the field pattern in the cavity. For the  $E_{011}$ ,  $E_{013}$  and the  $E_{015}$  modes the  $E_r$  component at the coating surface, which is proportional to  $\sin \theta_1$ , is very small. (See Figs. 5.7 - 5.11). A slight error in measuring the resonance frequency causes a large change in  $\sin \theta_1$  and hence in the calculation of the conductance. To show this effect the values of  $Q\sigma_1'$ , evaluated from equation 3.34 or 3.37, were calculated for different values of frequencies near the resonance of each mode. These values are shown in Table 5.9. From these calculations it is clear that only the results for the  $E_{012}$  and  $E_{014}$  modes are reliable.

The results obtained for the disk coated on only one side indicate that the field for the odd modes is not symmetric. The reason for this may be that, since the  $E_r$  field at the coating is near to zero, the coating may divide the cavity into two resonating portions. This is



| Coating     | Frequency | $\Delta f$ M Hz | Q    | $10^4/Q$ | $10^4/Q_c$ |
|-------------|-----------|-----------------|------|----------|------------|
| None        | 5.739     | 0.4             | 7200 | 1.4      |            |
| 200 Å Plat. | 5.739     | 0.5             | 5740 | 1.74     | 0.34       |
| 100 Å Al.   | 5.739     | 0.642           | 4470 | 2.23     | 0.83       |
| 200 Å Al.   | 5.737     | 5.2             | 550  | 18.2     | 16.8       |
| Carbon #1   | 5.739     | 0.92            | 3100 | 3.23     | 1.83       |
| Carbon #2   | 5.738     | 0.75            | 3820 | 2.61     | 1.21       |

Table 5.3 Q values for the  $E_{011}$  mode.

| Coating     | Frequency             | $\Delta f$ M Hz | Q    | $10^4/Q$ | $10^4/Q_c$ |
|-------------|-----------------------|-----------------|------|----------|------------|
| None        | 5.9012                | 0.66            | 4370 | 2.29     |            |
| 200 Å Plat. | 5.906                 | 0.92            | 3200 | 3.13     | 0.84       |
| 100 Å Al.   | 5.901                 | 9.2             | 321  | 31.1     | 28.81      |
| 200 Å Al.   | Could not be measured |                 |      |          |            |
| Carbon #1   | 5.905                 | 3.0             | 983  | 10.18    | 7.89       |
| Carbon #2   | 5.8993                | 3.8             | 775  | 12.9     | 10.61      |

Table 5.4 Q values for the  $E_{012}$  mode.

| Coating     | Frequency | $\Delta f$ M Hz | Q    | $10^4/Q$ | $10^4/Q_c$ |
|-------------|-----------|-----------------|------|----------|------------|
| None        | 6.13      | 0.525           | 5850 | 1.71     |            |
| 200 Å Plat. | 6.129     | 0.6             | 5100 | 1.96     | 0.15       |
| 100 Å Al.   | 6.13      | 0.7             | 4380 | 2.28     | 0.57       |
| 200 Å Al.   | 6.13      | 2.0             | 1530 | 6.54     | 4.83       |
| Carbon #1   | 6.129     | 1.09            | 2820 | 3.54     | 1.83       |
| Carbon #2   | 6.128     | 1.15            | 2664 | 3.75     | 2.04       |

Table 5.5 Q values for the  $E_{013}$  mode.



| Coating                  | Frequency             | $\Delta f$ M Hz | Q    | $10^4/Q$ | $10^4/Q_c$ |
|--------------------------|-----------------------|-----------------|------|----------|------------|
| None                     | 6.72                  | .94             | 3580 | 2.71     |            |
| 200 Å Plat.              | 6.673                 | 1.05            | 3170 | 3.13     | .42        |
| 100 Å Al.<br>200 Å Al. } | Could not be measured |                 |      |          |            |
| Carbon #1                | 6.67                  | 7.2             | 465  | 21.56    | 18.85      |
| Carbon #2                | 6.669                 | 9.4             | 354  | 28.2     | 25.5       |

Table 5.6 Q values for the  $E_{014}$  mode.

| Coating     | Frequency | $\Delta f$ M Hz | Q    | $10^4/Q$ | $10^4/Q_c$ |
|-------------|-----------|-----------------|------|----------|------------|
| None        | 7.142     | 0.45            | 7900 | 1.26     |            |
| 200 Å Plat. | 7.14      | 0.53            | 6730 | 1.49     | 0.23       |
| 100 Å Al.   | 7.14      | 0.72            | 4950 | 2.02     | 0.96       |
| 200 Å Al.   | 7.14      | 0.9             | 3960 | 2.52     | 1.26       |
| Carbon #1   | 7.141     | 0.75            | 4780 | 2.09     | 0.83       |
| Carbon #2   | 7.142     | 0.72            | 4950 | 2.02     | 0.76       |

Table 5.7 Q values for the  $E_{015}$  mode.

| Metal         | 200 Å Plat. | 100 Å Al. | 200 Å Al. | Carbon #1 | Carbon #2 |
|---------------|-------------|-----------|-----------|-----------|-----------|
| $\sigma$ Mhos | 0.184       | 0.372     | 0.744     | Unknown   | Unknown   |
| $E_{011}$     | 0.00299     | 0.0073    | 0.147     | 0.0161    | 0.0106    |
| $E_{012}$     | 0.000225    | 0.000774  |           | 0.000166  | 0.000222  |
| $E_{013}$     | 0.0025      | 0.00855   | 0.0725    | 0.032     | 0.0306    |
| $E_{014}$     | 0.000612    |           |           | 0.000142  | 0.000193  |
| $E_{015}$     | 0.00107     | 0.0045    | 0.00586   | 0.00286   | 0.00354   |

Table 5.8 Measured conductance.





| Mode      | Frequency GHz | $Q \sigma_1'$       |
|-----------|---------------|---------------------|
| $E_{011}$ | 5.742         | $1.247 \times 10^4$ |
|           | 5.745         | $2.151 \times 10^2$ |
|           | 5.747         | $7.033 \times 10$   |
| $E_{012}$ | 5.865         | .275                |
|           | 5.88          | .246                |
|           | 5.91          | .235                |
|           | 5.96          | .253                |
| $E_{013}$ | 6.12          | $8.12 \times 10$    |
|           | 6.127         | $7.365 \times 10^3$ |
|           | 6.134         | $1.55 \times 10^2$  |
|           | 6.135         | $1.28 \times 10^2$  |
| $E_{014}$ | 6.6           | .0727               |
|           | 6.675         | .0732               |
|           | 6.72          | .0748               |
| $E_{015}$ | 7.14          | $1.157 \times 10$   |
|           | 7.155         | $7.0155 \times 10$  |
|           | 7.1625        | $1.383 \times 10^3$ |
|           | 7.2           | 6.09                |

Table 5.9



shown very clearly in the case of the  $E_{013}$  mode where a resonance curve with two peaks was observed.

This effect may also occur for the case of disks coated on both sides. In this case the films may divide the cavity into three resonating portions which could be another reason for the inconsistency of the results.

For the antisymmetric modes the  $E_r$  field is maximum at the coating surface and thus it is not possible that any portion of the cavity may resonate on its own.



## CONCLUSIONS

For a periodically loaded cylindrical waveguide, the relations between the phase shift per section and the waveguide dimensions were derived for the cases where the loading disk consists of up to three layers of different dielectric material. Both the  $E_{m,n}$  and the  $H_{m,n}$  modes were considered.

The case when the loading element is a dielectric disk coated with a thin layer of a nonmagnetic metal was analysed. Two formulas, which relate the phase shift and the attenuation per section to the waveguide dimensions and the coating conductivity were derived. The phase shift and the attenuation per section were plotted against the frequency for two different structures and for different coating conductivity. It was found that the attenuation per section varies with the phase shift and it is not the same for the different pass bands. The dispersion curves show a change in shape in the vicinity of the stop bands; the group velocity at the  $\pi$  mode is no longer equal zero.

A cavity loaded with a dielectric disk was used to determine the conductance of the coating by measuring the  $Q$  values for different resonating modes. The cavity used in this work was found to be suitable for measuring a coating of resistance not lower than  $5 \text{ K } \Omega$ . For lower coating resistance the losses are high and the  $Q$  values cannot be measured with the same set-up. Also for very high values of resistance the effect of the coating on the  $Q$  values is very small and hence an accurate result



cannot be calculated.

It is expected that the measured conductance is accurate to 10%. The main sources of error are in measurement of the bandwidth. In setting the pips at the 3dB points an error of about 5% is possible. Determination of the 3dB line using the attenuator and the loading effect of the probes cause an additional errors which are expected to be relatively small.

It is possible to measure lower coating resistance using another cavity by placing the coating in a region with low electric field intensity,  $E_r$ . From the discussion of chapter 5, it is clear that the low value of  $E_r$  must be chosen so that the air space phase shift,  $\theta_1$ , varies little with the frequency. This can be done by making  $\theta_1$  less than  $90^\circ$  and by using a frequency well above cut-off and by avoiding the nulls of  $E_r$ .

The attenuation introduced into a structure by using coated disks depends on the structure dimensions. For a coating resistance of  $5K\Omega$  and for the two structures discussed in chapter 2, the attenuation per section was calculated. For the  $\frac{\pi}{2}$  mode it was found to be about 10.9 dB when  $\frac{p}{a} = 7$ ,  $\frac{q}{a} = .12$  and 0.0261 dB when  $\frac{p}{a} = 0.1$ ,  $\frac{q}{a} = 0.1$ . In both of these cases the  $\frac{\pi}{2}$  mode occurs below cut-off and the large difference in attenuation is mainly due to the difference in  $\frac{p}{a}$ . This can be seen from equation 2.14 which shows that  $\alpha$  must increase with  $\sqrt{p}$ .





In a travelling wave tube the smaller value of  $\frac{p}{a}$  is more typical so that the attenuation for a 5 K $\Omega$  coating is approximately .03 dB/section. To obtain a useful amount of attenuation, say 3 dB per section, then the coating resistance must be considerably reduced. As already stated, another cavity must be designed in order to measure the lower values required.

Coated disks can be used not only to perform isolation between the input and the output in a travelling wave tube, if a dielectric loaded structure is used, but also to suppress the dominant mode ( $H_{11}$ ). Since any H mode in a circular waveguide has two transverse electric field components ( $E_r$  and  $E_\phi$ ), the losses due to the coating are expected to be higher than that of the  $E_{01}$  mode. For the same reason a lossy coating could be used to suppress unwanted modes in a linear accelerator.

A metallic coating can also be used to reduce the build up of electrostatic charge, and thus increase the surface breakdown strength in applications where high electric field intensities occur e.g. microwave windows<sup>(12)</sup>.



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## APPENDIX

$$\begin{aligned}
 \int_{-\frac{q}{2}}^{\frac{q}{2}} \sin^2 \beta_d z dz &= \frac{1}{2} \int_{-\frac{q}{2}}^{\frac{q}{2}} (1 - \cos 2\beta_d z) dz \\
 &= \frac{1}{2} \left[ z - \frac{\sin 2\beta_d z}{2\beta_d} \right]_{-\frac{q}{2}}^{\frac{q}{2}} \\
 &= \frac{1}{2} \left( q - \frac{\sin \beta_d q}{\beta_d} \right)
 \end{aligned}$$

..... A.1

$$\begin{aligned}
 \int_{-\frac{q}{2}}^{\frac{q}{2}} \cos^2 \beta_d z dz &= \frac{1}{2} \int_{-\frac{q}{2}}^{\frac{q}{2}} (1 + \cos 2\beta_d z) dz \\
 &= \frac{1}{2} \left[ \left( z + \frac{\sin 2\beta_d z}{2\beta_d} \right) dz \right]_{-\frac{q}{2}}^{\frac{q}{2}} \\
 &= \frac{1}{2} \left( q + \frac{\sin \beta_d q}{\beta_d} \right)
 \end{aligned}$$

..... A.2

$$\begin{aligned}
 \int_{\frac{q}{2}}^{\frac{1}{2}} \cos^2 \beta_a \left( z - \frac{1}{2} \right) dz &= \frac{1}{2} \int_{\frac{q}{2}}^{\frac{1}{2}} \left( 1 + \cos 2\beta_a \left( z - \frac{1}{2} \right) \right) dz \\
 &= \frac{1}{2} \left[ z + \frac{\sin 2\beta_a \left( z - \frac{1}{2} \right)}{2\beta_a} \right]_{\frac{q}{2}}^{\frac{1}{2}} \\
 &= \frac{1}{4} \left( 1 - q + \frac{\sin \beta_a (1-q)}{\beta_a} \right)
 \end{aligned}$$

..... A.3





$$\begin{aligned}
 \int_0^a J_1^2(\chi r) r dr &= \left[ \frac{r^2}{2} (J_1^2(\chi r) - J_0(\chi r) J_2(\chi r)) \right]_0^a \\
 &= \left[ \frac{a^2}{2} (J_1^2(\chi a) - J_0(\chi a) J_2(\chi a) \right. \\
 &\quad \left. + J_0(0) J_2(0)) \right]
 \end{aligned}$$

$$\text{but } J_0(\chi a) = J_2(0) = 0$$

$$\therefore \int_0^a J_1^2(\chi r) r dr = \frac{a^2}{2} J_1^2(\chi a) \quad \dots \text{A.4}$$

$$\int_0^a J_0^2(\chi r) r dr = \frac{a^2}{2} J_1^2(\chi a) \quad \dots \text{A.5}^{(5)}$$















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